Catalog Description

Basic concepts of differentiation and integration with applications. Integrals of trigonometric, exponential and logarithmic functions; differential equations; techniques of integration and applications, infinite series. Pre: C or better in Math 104 or Math 104G, or Math Placement Score greater than 49 for enrollment in Math 205; C or better in Math 205, or Math Placement Score greater than 59 for enrollment in Math 206.

It should be noted that both Math 205 and 206 include 3-credit lecture and 1-credit computer-based lab components. The outline provided below is for the lecture portion of the course. The lab portion of the course is designed to enhance the concepts presented in the lecture by providing numerical representations of the concepts. The primary goals of the lab are to introduce the students to computer based algebra systems (CAS) and other available information technologies (e.g. spreadsheets, graphing calculators, etc.), to understand when the use of computer technology is appropriate and when it is not, to provide numerically based solutions to Calculus problems, and most importantly to enhance their understanding of the Calculus through computer based visualization capabilities. For certain topics (e.g. differential equations), the lecture instructor may choose for the topic to be covered entirely within the lab, but for most topics the intent is for the lab to proceed closely with the lecture, for the lab to enhance students’ understanding of the lecture material by presenting the same material from a numerical approximation perspective. The final grade for each course is weighted 75% lecture and 25% lab, with the lab instructor providing lab component feedback to the lecture instructor for determination of the final grade, which is determined by the lecture instructor.

Course Goals/Learning Outcomes Objectives for Math 205

The primary goals of Math 205 include providing students an introduction and then a strong background in the fundamental concepts and techniques associated with first semester Calculus, namely the Derivative and the Definite Integral. Math 205 is a prerequisite for Math 206, so an ancillary but important goal is to prepare students for success in Math 206. Care should be taken to ensure that students gain both a deep understanding of these concepts as well as becoming proficient in the associated techniques usually used to solve the Calculus problems. University mathematics typically places more emphasis on gaining a true understanding of concepts than students might have been exposed to in High School mathematics courses, and consequently an additional goal of Math 205 is increase students’ mathematical sophistication.

Students successfully completing the course should be able to:
- Compute limits both graphically and symbolically
- Determine if a function is continuous at each point
- Compute the derivative at a point using the definition
- Become proficient in using all rules of differentiation (see below) to compute global derivatives, and knowing when and when not to use each rule
- Apply the conceptual derivative to linear approximation
- Utilize the symbolic derivative to find the equation for a tangent line and use that line to [linearly] approximate functional values
- Explain in practical terms the concept of the derivative
- Utilize implicit differentiation to solve related rates problems
- Utilize the derivative to optimize functions
- Utilize the derivative and higher order derivatives to graph functions
- Understand and apply the derivative and higher order derivatives to modeling problems
- Utilize L’Hospital’s Rule and know how to deal with the different indeterminate forms
- Understand and apply the Fundamental Theorem of Calculus
- Use the definition of the Definite Integral to set up problem solving models
- Understand and apply Integration as a process (see note on the Definite Integral below)
- Anti-differentiate basic functions and utilize u-substitution where appropriate

**Topics Covered**
- Pre-Calculus Review (optional, but recommended)
- Limits and Continuity
- The Derivative
- Rules of Differentiation
- Applications of the Derivative
- The Definite Integral

**Pre-Calculus Review (1-2 weeks)**

See outline for Math 104, with particular emphasis on:

- Graphical Modeling and Interpretation
- Function Domains
- The fundamental properties of the exponential and log functions
- Unit Circle Trigonometry

**Limits and Continuity (2 weeks)**

Conceptual – instructors need to be careful to ensure that students truly understand the concept of limit, as opposed to simply performing a lot of algebra and not being able to interpret their results. This is not to say that students do not need to be able to perform algebra, but algebra for the sake of algebra will only improve their algebraic skills.

- Graphical interpretation of limits (students should be able to identify limits, if they exist, or explain why they do not, without the use of formulas)
- Overall purpose of the concept of limit (e.g. $1 = .9$, defining pi, adding infinitely many numbers, instantaneous velocity and rate of change)
- Graphical left/right-hand limits
- Limits at infinity (utilize the concept of certain functions “growing” faster than others)
- Understand and apply the connection between limits and continuity

Skills
- Compute limits algebraically using basic algebraic techniques
- Compute limits at infinity using algebraic techniques
- Determine if piecewise defined functions are continuous using symbolic manipulation

The Derivative (3 weeks)

Conceptual
- Definition of the Derivative
- Instantaneous velocity and rate of change/slope of the tangent line
- Explain in practical terms the statement that $C'(100) = 10$

Skills
- Compute the Derivative of a function at a point, given symbolically, using the definition
- Find the equation for the tangent line to a function at a point
- Compute an approximation to the Derivative of a function given numerically, at a single point

Rules of Differentiation (3 weeks)

Skills
- Power Rule, Product Rule, Quotient Rule, Chain Rule, Exponential and Logarithmic Derivatives, Trigonometric Derivatives, Logarithmic Differentiation
- Computing higher order derivatives

Applications of the Derivative (3 weeks)

Conceptual
- Linear approximation from a conceptual view: Given $C(100) = 2000$ and $C'(100) = 10$, approximate $C(101)$.
- Related Rates: ideally, students can use the concept of the derivative to approximate solutions to related rates problems (optional)
- Apply the concept of the derivative and second derivative to sketch graphs
- Given the graph of the derivative of $f$, sketch a possible function $f$
- Conceptual understanding of why L’Hospital’s Rule works
- Given graphs of $f$, $f'$, and $f''$ determine which is which
- Given a standard position function, utilize the derivative to understand movement
- Given the graph of the derivative of position, answer modeling questions regarding maximum speed, acceleration, and position

Skills

- Linear Approximation: given a symbolic formula for f, approximate the value for f near a value for x at which f’s value and that of its derivative are easily computed symbolically (or given numerically via a table of values for example)
- Implicit Differentiation
- Related Rates – solve standard related rates problems
- Optimization – solve standard optimization problems
- Compute critical numbers from symbolically defined functions and use the symbolic derivative and second derivative to graph f
- Utilize L’Hospital’s Rule to compute indeterminate form limits

**Definition of the Definite Integral (3 weeks)**

Note: one of the primary goals of this portion of Math 205/206 is to get the students to truly understand the concept of “integration” as a process, the process of dividing things into small pieces, finding out information about each piece, and then summing up the results. It is only if they understand the “process” that they might stand a chance of tackling a non-traditional type of problem in the future.

**Fundamental Theorem of Calculus**

Conceptual
- Definition of the definite integral as a Riemann limit
- “area under” velocity = change in position, “area under” f’ = change in f
- If you wish to find area under f, find a function F such that F’ = f and evaluate F(b) – F(a)
- If you wish to find change in f, evaluate the definite integral for f’ (either symbolically, graphically, or numerically, e.g. with your calculator). It is essential that the student knows when it is appropriate to use which
- Understand the fact that every continuous function over [0,1], for example, has an antiderivative (namely the definite integral over [0,x])

Skills (Anti-differentiation)

- \( \int_{a}^{b} f'(x) \, dx = f(b) - f(a) \) (Actually computing the value symbolically)
- Algebraic simplification (reducing to the sum of powers)
- Standard functions (polynomial, basic trig., basic exp., ln)
- U-Substitution
- Guess and Check (optional)
- Using the fact that every continuous function over [0,1], for example, has an antiderivative (namely the definite integral over [0,x]), use the chain rule to compute definite integrals of the form \([0,g(x)]\). (optional)

**Numerical Integration (optional – generally covered in the lab)**

Conceptual
- Approximating Definite Integrals using Left(n), Right(n), Trap(n), Mid(n), and Simp(n), and knowing under which conditions can you determine if each approximation is too large, too small, or indeterminable. (Note: Simp(n) can be represented as a weighted average between Trap and Mid…Simp(n) = (Trap(n) + 2Mid(n))/3)
- Given that f is monotonic, find the least n such that Left(n) is within $\varepsilon$ of $\int_a^b f$
- Conclusions that can be made regarding these approximations, given concavity (e.g. if f is monotonic and concave up over $[a,b]$, then Trap(n) is too big, etc.)

Skills – covered in lab
Course Goals/Learning Outcomes Objectives for Math 206

Math 206 is a continuation of Math 205. It utilizes the derivative knowledge and introduction to the definite integral gained in Math 205 to build a strong understanding of integration, both as a process and a tool, and how to use integration to solve volume and other application problems. Once techniques of anti-differentiation are learned we then extend that knowledge to an introduction to differential equations and finally come full circle back to the derivative/approximations to Taylor Polynomials.

Students successfully completing the course should be able to:

- Understand and apply the Fundamental Theorem of Calculus
- Use the definition of the Definite Integral to set up problem solving models
- Understand and apply Integration as a process (see note on the Definite Integral below)
- Use the Definite Integral to find volumes by the washer/shell/slicing methods
- Use the Definite Integral to solve work problems
- Identify appropriate methods of anti-differentiation (i.e. u-sub, parts, trig, trig sub) and be able to apply the methods
- Solve differential equations from slope fields, identify equilibrium solutions and stability, apply Euler’s Method, separation of variables, and by multiplying through by an integrating factor (optional, covered in the lab)
- Identify whether or not a series converges utilizing various tests (e.g. terms must go to zero, alternating series, absolute convergence, root/ratio test, comparison)
- Identify certain types of series (e.g. geometric, harmonic) and be able to compute the values for geometric series
- Compute radius and interval of convergence for power series
- Compute [at least] the first few values for a Taylor Polynomial
- Compute and manipulate McLauren Series for sin, cos, exp

Note: There is the definite possibility of overlap between the material at the end of Math 205 and the beginning of Math 206. The amount of overlap beginning in Math 206 is entirely up to the instructor. While minimal review of the definition of the Definite Integral is certainly warranted in all cases, some instructors may choose to cover the subject again in great detail, which is fine provided adequate time is left for subsequent topics.

Topics Covered

- Definition of the Definite Integral
- Applications of the Definite Integral
- Antidifferentiation Techniques
- Differential Equations
- Infinite Series
The times allotted for each subject are of course only suggestions and as such should be used only as guidelines. Some instructors do not cover much on the definition and spend more time on applications and anti-differentiation, and some instructors skip differential equations in the lecture portion of the class. Care should be taken to allow enough time for covering Taylor Series in the infinite series portion of the course.

**Definition of the Definite Integral (3 weeks)**

Note: one of the primary goals of this portion of Math 205/206 is to get the students to truly understand the concept of “integration” as a process, the process of dividing things into small pieces, finding out information about each piece, and then summing up the results. It is only if they understand the “process” that they might stand a chance of tackling a non-traditional type of problem in the future.

**Fundamental Theorem of Calculus**

- **Conceptual**
  - Definition of the definite integral as a Riemann limit
  - “area under” velocity = change in position, “area under” $f' = \text{change in } f$
  - If you wish to find area under $f$, find a function $F$ such that $F' = f$ and evaluate $F(b) - F(a)$
  - If you wish to find change in $f$, evaluate the definite integral for $f'$ (either symbolically, graphically, or numerically, e.g. with your calculator). It is essential that the student knows when it is appropriate to use which
  - Understand the fact that every continuous function over $[0, 1]$, for example, has an antiderivative (namely the definite integral over $[0, x]$)

**Skills (Anti-differentiation)**

- $\int_{a}^{b} f' = f(b) - f(a)$ (Actually computing the value symbolically)
- Algebraic simplification (reducing to the sum of powers)
- Standard functions (polynomial, basic trig., basic exp., ln)
- U-Substitution
- Guess and Check (optional)
- Using the fact that every continuous function over $[0, 1]$, for example, has an antiderivative (namely the definite integral over $[0, x]$), use the chain rule to compute definite integrals of the form $\int_{0}^{g(x)}$. (optional)

**Numerical Integration (optional – generally covered in the lab)**

- **Conceptual**
  - Approximating Definite Integrals using Left(n), Right(n), Trap(n), Mid(n), and Simp(n), and knowing under which conditions can you determine if each approximation is too large, too small, or indeterminable. (Note: Simp(n) can be represented as a weighted average between Trap and Mid…Simp(n) = $\frac{(\text{Trap(n)} + 2\text{Mid(n)})}{3}$)
  - Given that $f$ is monotonic, find the least $n$ such that Left(n) is within $\varepsilon$ of $\int_{a}^{b} f$
Conclusions that can be made regarding these approximations, given concavity (e.g. if $f$ is monotonic and concave up over $[a,b]$, then $\text{Trap}(n)$ is too big, etc.)

Skills – covered in lab

Applications of the Definite Integral (3 weeks)

Note: emphasis here should be on setting up the definite integrals...again, towards an understanding of the integration process. Computing the values is covered in the Definite Integral above. A useful technique in this regards might be to emphasize that setting up the Riemann limit, which then according to the definition yields an integral, requires nothing more than identifying the value for one single washer/shell/slice/arc-length/work/pressure, etc.

Conceptual
- Volumes of Revolution (washer and shell methods)
- Volumes by slicing
- Arc Length (one might want to emphasize here that despite the fact that the “obvious” method of finding arc length does not yield a definite integral according to the Riemann definition above without first applying an algebraic trick along with continuity of the first derivative, utilizing a computer to get an approximation would usually yield the result and hence is a completely reasonable approach)
- Work (primarily, $\text{work} = \text{weight} \times \text{distance moved}$)
- Optional (Pressure, Center of Mass, Probability, etc.)

Skills
- Visualizing solids of revolution, 3-dimensional objects in general, work, etc.
- Finding points of intersection and determining limits of integration
- Review of simple anti-derivatives and computing the definite integral (Optional – since the emphasis here is setting up the integrals, computing them can be done via technology if the instructor wishes)

Antidifferentiation Techniques (3+ weeks)

Emphasis here is to learn the basic techniques, and is therefore primarily skill oriented. Many instructors find it useful to assign take-home work in order to allow the students the necessary time to become proficient without spending too much time in class. Again, this is entirely at the discretion of the instructor.

- $u$-sub
- parts
  - standard parts (including tabular parts)
  - circular parts
  - parts applied to ln, inverse trig, etc.
- basic trigonometric anti-differentiation (e.g. $\sin^3 x \cos x$, $\sin^3 x$, $\tan x \sec^2 x$, etc.)
- trigonometric substitutions ($x$-sub)
- improper integrals (some instructors prefer to cover this section when they cover infinite series)
- partial fractions (optional, covered in the lab)

**Differential Equations (2 weeks)**

Note: this topic is optional in the sense that some instructors prefer to cover the material in the lab. When covered in class it is mostly conceptual, with a little skill intermixed.

**Conceptual**
- Slope fields (both drawing and utilizing to approximate solutions)
- Equilibrium solutions for \( y' = f(x, y) \) (Set \( y' = 0 \), check if you get a constant solution (i.e. \( y = c \))
  - Stability (check visually from slope field, and numerically by asking if \( y < c \), is \( y' > 0 \), causing \( y \) to return to \( c \), etc.)

**Skills**
- Determine whether \( f \) is a solution to a differential equation (simply take the necessary derivatives and "plug them into the appropriate slots")
- Euler’s Method
- Solving separable equations
- Exact equations (Optional)

**Infinite Series (5 weeks)**

Note: The ultimate goal here is to gain an understanding of power series and more importantly Taylor Polynomials. All too often time runs out and the students only think of infinite series in terms of the convergence rules and techniques, with emphasis only on specific series and not series-defined functions. So, care must be given to allow enough time.

The study of infinite series intermingles the conceptual and the necessary skills throughout. What follows is a useful outline on the subject.

**General Convergence of** \( \sum a_n \): 
- Is \( \lim_{n \to \infty} a_n = 0 \)? (Necessary, but not sufficient. In other words, if yes, it might converge; if no, then it definitely does not. You should always check this FIRST.)
- Is it an alternating series? (If yes, then combined with a above, it converges.)

**Recognizing Series:**
- Harmonic Series \( \sum \frac{1}{n} \) diverges.
- Geometric Series \( \sum r^n \) (converges if \(|r| < 1\), diverges otherwise)
- P-series \( \sum \frac{1}{n^p} \) (converges if \(|p| > 1\), diverges otherwise)

**Computing the value of series:**
Generally this is not possible. But, for certain types of series, like telescoping (optional) and geometric, it is possible. You should know the trick for finding the partial sum for geometric series, and then for computing the value (of course the value is always \( \frac{1}{1 - r} \) if the series begins with a 1).

Tests for convergence (not for finding the sum, just whether it exists or not)

- Comparison (are the terms of the series less than a series that you already know converges, such as a geometric or \( p \)-series? If yes, then the series converges. Are the terms of the series greater than the terms of a divergent series such as the harmonic, or a geometric or \( p \)-series that diverges? If yes, then the series diverges.)
- Integral Test (optional)
- Root/Ratio tests (some students benefit from seeing that the reason these seemingly abstract tests work is that they easily compare to geometric series)

Power Series

- Find the radius of convergence (usually this entails finding the values of \( x \) that would work for the \( r \) in the geometric series, \( p \)-series, or root/ratio tests).
- Find the interval of convergence (just check the endpoints from \( j \) above)

Taylor/Maclaurin Series

- Know the purpose of investigating Taylor Polynomials (Here students should understand that the easiest types of functions for us to investigate are polynomials, as they are algebraic and easily computed. The next best thing for a non-polynomial, such as \( \sin \) or \( \exp \) is to be able to represent them as “infinite polynomials” which we refer to as power series or when applied to a given known function such as \( \sin \), as Taylor Polynomials
- Know the general form for the Taylor Polynomial
- Know the Taylor Polynomials for the exponential, \( \sin \), \( \cos \).
- Be able to determine how many terms are necessary (i.e. which “finite” polynomial can be used to) to approximate \( \sin \) (close to zero for example) to within a desired pre-determined error.
- Be able to compute the Taylor Polynomial for general functions (or at least the first few coefficients).
- Check if a series is a Taylor Polynomial for a given function. (Note: students should know that the Taylor Polynomial for a convergent power series is itself, and as such each convergent power series is a Taylor Polynomial. But they should be able to compute the Taylor Polynomial for other functions as well.)
- Differentiate or integrate a Taylor Polynomial that is known, to get other Taylor Polynomials that would be difficult to compute term by term.