Program Review 2012
Mathematics Department Assessment Efforts

Introduction

The Mathematics Department has engaged in multiple assessment efforts since the last program review, including efforts in its major program and its General Education/Service mission. As part of the 2008 campus-wide assessment effort the Department developed student learning outcomes for its major program for both the traditional and teaching tracks, and developed a curriculum matrix that aligned those outcomes with its courses. At the same time the department developed a schedule for assessing the various outcomes throughout subsequent years, usually choosing two – three outcomes per year. Unfortunately, the department did not follow the initial assessment schedule. Recent developments have also required departments to assess institutional General Education goals. Fortunately, there is overlap in these two types of assessments. Our Calculus classes, for example, are an integral part of our major program, but are also part of the GenEd program and service numerous NS departments and Pharmacy. Thus, in many instances assessments for the program also gave us information regarding how well we are meeting institutional goals. The department engaged in eight assessment activities in recent years, four of which spanned more than one semester and one that spanned six semesters.

What follows is a detailed report of all assessment efforts since the last program review. Additionally, it is of particular interest to note that the department is striving to make assessment a more intuitive part of our work, involve a higher percentage of the faculty, and encourage more discussion of how well we are meeting our mission. To that end the department has adopted a user-friendly annual assessment report form that is used to identify two assessments each year and the faculty performing the assessments, and makes the reporting process and resulting discussions easier, and it has created a more recent assessment plan with complete department participation. It is the hope that such a form and plan will not only encourage broader participation, spreading the assessment load, but will help to departmentalize assessment as a natural part of what we do.

Glossary of abbreviations:

SLO – Student Learning Outcomes (in general)
CLO – Course Learning Outcomes (SLO’s from specific courses)
ILO – Institutional Learning Outcomes (Appendix A-Institutional Learning Outcomes)
PLO – [Math Department] Program Learning Outcomes (Appendix A – Curriculum Matrix)

---

1 See Appendix A - Curriculum Matrix.
2 See Appendix A - Math Department Major Assessment Timeline.
3 See Appendix A - Annual Assessment Report Form.
Summary of assessment efforts

All of the assessments except the one related to Analysis (math 431-432) were direct assessments that embedded problems that address specific learning outcomes or institutional goals directly into course exams. At least two faculty members met in each case to discuss the types of problems that should be assessed and the associated learning outcomes, and rubrics were jointly determined prior to scoring. In most instances, however, very little whole-department discussion occurred as a result of the assessments. This is a reflection of the department’s indifferent “state of assessment readiness”, which is currently only at an emerging level if viewed from the perspective of developing a “culture of assessment” as desired by WASC. Consequently, although the assessments were perhaps valuable to the faculty involved, who then used the results to “close the loop” and make adjustments to their teaching, the department is still struggling implementing methods that would close the loop on a broader basis.

Each of the assessment efforts provided valuable information about how the department is reaching its learning outcomes goals, particularly within certain courses. None of the assessments identified critical weaknesses, which given our record of teaching excellence is not entirely unexpected. The most useful assessments identified weaknesses in student understanding, usually in terms of the Analysis and Critical Thinking Institutional Goals and sometimes with respect to Program Learning Outcomes. The most detailed information of this type was gained in Math 206 and Math 311, Calculus II and Linear Algebra, respectively, which the department considers critical courses in terms of both the major and in serving Natural Sciences. The instructor(s) closed the loop in these two courses by addressing these weaknesses. The scores over multiple semesters showed marked improvement in some cases.
Compilation of Assessments since the last Program Review

<table>
<thead>
<tr>
<th>Timeframe</th>
<th>Faculty Involved</th>
<th>Classes Involved</th>
<th>SLO’s Assessed and Assessment Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>F 2011</td>
<td>Brian Wissman, Shuguang Li, Zorana Lazarevic, Mitchell Anderson</td>
<td>Math 205</td>
<td>QR ILO’s (Computation) Embedded Assessment</td>
</tr>
<tr>
<td>F 2008 and F 2011</td>
<td>Efren Ruiz, Mitchell Anderson</td>
<td>Math 205</td>
<td>QR ILO’s (Computation) Gateway Derivative Exams</td>
</tr>
<tr>
<td>F 09, Sp 10 (2), Su 11, F 11, Sp 12, Su 12, Sp 12</td>
<td>Mitchell Anderson, Brian Wissman</td>
<td>Math 206</td>
<td>QR ILO’s (all 3 and Critical Thinking) Embedded Assessment</td>
</tr>
<tr>
<td>Sp 2012</td>
<td>Brian Wissman, Mitchell Anderson, Shuguang Li</td>
<td>Math 206</td>
<td>QR ILO’s (all 3 &amp; Critical Thinking), Comm ILO’s (Organization &amp; Structure, Line of Reasoning) Group Project</td>
</tr>
<tr>
<td>Sp 2012</td>
<td>Brian Wissman, Mitchell Anderson</td>
<td>Math 206</td>
<td>QR ILO’s (all 3 and Critical Thinking) Embedded Assessment</td>
</tr>
<tr>
<td>Sp 2010, Sp 2012</td>
<td>Mitchell Anderson, Brian Wissman</td>
<td>Math 311</td>
<td>QR ILO’s (all 3 and Critical Thinking) PLO’s – All Embedded Assessment</td>
</tr>
<tr>
<td>F 2010 – Sp 2011</td>
<td>Mitchell Anderson, Brian Wissman, Roberto Pelayo, Efren Ruiz</td>
<td>Math 431-432</td>
<td>QR ILO’s (Analysis and Critical Thinking) PLO’s – All Course Portfolio</td>
</tr>
</tbody>
</table>
The details of each assessment are provided using a common template developed specifically for this purpose.

**Math 104 Assessment**

1. **Timeframe for the assessment(s).**
   
   ➢ Spring 2011, Spring 2012.

2. **Faculty involved.**
   
   ➢ Diana Webb and Zorana Lazarevic administered the assessment for Spring 2011, Diana Webb and Efren Ruiz administered the assessment for Spring 2012, and Mitchell Anderson provided oversight and assistance for both.

3. **Student Learning Outcomes Assessed**
   
   ➢ All Quantitative Reasoning ILO’s, including critical thinking.
   ➢ The following CLO’s
     - Apply function notation, particularly for the composition of functions
     - Recognize the standard functions and be able to recognize what sets each apart from others (polynomials for this assessment)
     - Solve [exponential and] logarithmic equations
     - Solve triangles
     - Solve trigonometric equations

4. **Courses in which the assessment was administered.**
   
   ➢ Math 104F, Math 104G, and Math 104

5. **Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.**
   
   ➢ Students in these courses are typically freshmen or sophomore NS majors.
   ➢ For Spring 2010, 70 students from three sections and two instructors took the 104F portion of the assessment, and 51 students from two sections and a single instructor took the 104G portion of the assessment. 41 students took the Math 104F final exam, 22 students took the Math 104G final exam, and 29 students took the Math 104 final exam, which included both the Math 104F and Math 104G problems.

---

4 See Appendix A – Math 104 Course Outline
For Spring 2011, 64 students from three sections and two instructors took the 104F portion of the assessment, and 28 students from one section and a single instructor took the 104G portion of the assessment. Of the 64 students taking the Math 104F portion, 36 were enrolled in Math 104F and the remaining 28 were enrolled in Math 104. Those in Math 104 were the only students who took part in the Math 104G assessment.

6. Details of the Assessment

- What type of assessment was administered (Direct or Indirect), and how was the data collected?
  
  A direct assessment was employed for each semester by embedding common problems into the final exams for multiple sections of Math 104, 104F, and 104G. The same problems were used each year. Three problems dealing with functions were embedded into the Math 104F finals and two trigonometry problems were embedded into the Math 104G finals. Students in Math 104 received all five problems.

- How was the assessment developed?
  
  Diana and Zorana met to identify three problems from Math 104F material to be embedded in the Math 104F/104 final exams, and two problems from Math 104G to be embedded in the Math 104G/104 final exams. The 104F problems assessed students’ abilities to use function notation and composition, the behavior of polynomials represented in factored form with repeated roots included, and using the properties of the log function to solve log equations. The 104G problems assessed solving right triangles and solving a trig equation with a multiple of the variable x.

  The problems were embedded on the last page of the final exams to make their appearance uniform across instructors and sections, to better facilitate copying after the exams were turned in, and to help assure student anonymity (i.e. no names were present on the last page).

---

5 The Spring 2010 assessment was undertaken at the request of the UHH Assessment Support Committee and was intended as a pilot to test the potential for using the newly developed Quantitative Reasoning and Scientific Inquiry institutional goals (Appendix A – Institutional Learning Objectives for Quantitative Reasoning).

6 Appendix - Math 104F Assessment Problems and Rubrics, and Math 104G Assessment Problems and Rubrics
The same problems were used for the second semester, Spring 2011.

- How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)

- Diana, Zorana, and Mitchell developed scoring rubrics for each problem by assigning points to various concepts and steps associated with each problem, with each test question receiving integer scores of between 0 and 4 for four of the problems and between 0 and 3 for the remaining problem. They then aligned each problem to one or more of the Quantitative Reasoning ILO’s (Calculations, Analysis, and Visual\(^7\)); each problem also corresponded to a course CLO (provided above). The numerical average percentage scores were then translated to achieving various levels for the ILO’s and CLO’s as designated in the following table:

<table>
<thead>
<tr>
<th>Raw Score</th>
<th>ILO or CLO equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 25%</td>
<td>Beginning</td>
</tr>
<tr>
<td>26% - 50%</td>
<td>Emerging</td>
</tr>
<tr>
<td>51% - 75%</td>
<td>Competent</td>
</tr>
<tr>
<td>76% - 100%</td>
<td>Advanced</td>
</tr>
</tbody>
</table>

- For each semester, Mitchell joined the faculty members to individually score each problem according to the rubrics. They then compared their results for each student and each problem, reconciling any differences. Averages of the totals were compiled. The same scoring rubric was used each semester.

7. Results and analysis.

One important result for this assessment was validation of the Quantitative Reasoning Institutional Goal, including the Critical Thinking component, as developed by the UHH Faculty Congress’ Assessment Support Committee. The Assessment Committee was grateful to have a working model that utilized the new institutional goals (ILO’s) and rubrics, and confidently submitted their work to the UHH Congress for approval. Mitchell sent a final report on the first semester pilot project to Dr. Seri Luangphinith, Chair of the Assessment Support Committee and it was later included in a WASC

\(^7\) Critical Thinking, another Institutional Goal that crosses over most other institutional goals, is also recognized as being satisfied by the Analysis and Visual components of Quantitative Reasoning and Scientific Inquiry. It is not treated separately within our rubric.
Resource Binder for use in their Assessment 101 workshop held at the Waikiki Beach Marriott in Honolulu, Feb. 2012.  

Results for the Spring 2010 104 Assessment

<table>
<thead>
<tr>
<th></th>
<th>Math 104F</th>
<th></th>
<th>Math 104G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>70 Exams</td>
<td></td>
<td>51 Exams</td>
</tr>
<tr>
<td>Possible Score</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Average</td>
<td>2.71</td>
<td>3.38</td>
<td>2.39</td>
</tr>
<tr>
<td>ILO’s Assessed</td>
<td>Calculations</td>
<td>Analysis and Visual</td>
<td>Calculations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculations</td>
<td>and Visual</td>
</tr>
<tr>
<td>CLO’s Assessed</td>
<td>Function</td>
<td>Properties of Polynomials</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Composition</td>
<td>Log Equations</td>
<td>Solve Trig Equations</td>
</tr>
<tr>
<td>Quantitative</td>
<td>Competent</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
<tr>
<td>Reasoning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Translation</td>
<td></td>
<td></td>
<td>Approaching</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Competent</td>
</tr>
</tbody>
</table>

It was interesting and perhaps gratifying to note that the students performed better on the conceptual material that required the use of visual critical thinking, regardless of whether or not the problem came from 104F material or from 104G. Students performed satisfactorily on all problems except G2, reaching or approaching competency. A discussion of G2 is provided below under closing the loop.

It should be noted that these are aggregate scores and that as expected the standard deviation for this particular data grew for lower scores, indicating that many students in that instance scored much higher than the mean. However, no deep statistical analysis was performed for the data.

---

Results for the Spring 2011 104 Assessment

<table>
<thead>
<tr>
<th></th>
<th>Math 104F 70 Exams</th>
<th>Math 104G 51 Exams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible Score</td>
<td>QI 4</td>
<td>QII 3</td>
</tr>
<tr>
<td></td>
<td>QIII 4</td>
<td>GI 4</td>
</tr>
<tr>
<td>Average</td>
<td>2.89</td>
<td>1.75</td>
</tr>
<tr>
<td>ILO’s Assessed</td>
<td>Calculations</td>
<td>Analysis and Visual</td>
</tr>
<tr>
<td></td>
<td>Calculations</td>
<td>Calculations</td>
</tr>
<tr>
<td></td>
<td>All competencies</td>
<td></td>
</tr>
<tr>
<td>CLO’s Assessed</td>
<td>Function Composition</td>
<td>Properties of Polynomials</td>
</tr>
<tr>
<td></td>
<td>Log Equations</td>
<td>Solve Triangles</td>
</tr>
<tr>
<td></td>
<td>Solve Trig Equations</td>
<td></td>
</tr>
<tr>
<td>Quantitative Reasoning</td>
<td>Competent</td>
<td>Approaching Competent</td>
</tr>
<tr>
<td>Translation</td>
<td></td>
<td>Approaching Competent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Advanced</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Approaching Competent</td>
</tr>
</tbody>
</table>

This second assessment compares favorably in some aspects to the first, and less so in other aspects. This variation is somewhat expected given the difference in instructors and their differing emphases, and of course student aptitude is also not uniform across classes. The results for function composition, a purely symbolic concept in this assessment, improved from 2.71 to 2.89, but the results for polynomials and log equations dropped significantly, almost 50% and 40% respectively. The Trigonometry results remained almost identical, which again is not surprising since the same instructor taught Trig for both studies. The results indicate a possible weakness in these two areas, properties of polynomial functions and solving logarithmic equations. See the discussion below on this matter in Closing the Loop.

8. Closing the loop.

The three faculty members involved in the Spring 2010 assessment discussed the results of that pilot. They acknowledged that the students performed satisfactorily in all but one area. With respect to problem G2, in which students were asked to solve a trigonometric equation involving a double angle (one of the last topics covered in Math 104 and one of the most difficult), the instructor noted that s/he intended to try to allot more time for that subject in the future.

The results on the first problem in 104F and both in 104G were very much the same the second time as the first. The trigonometry instructor did mention that time constraints did not allow the extra time hoped for after the first assessment to spend more on the final problem; with [not surprisingly] virtually identical scores as a result. The biggest surprise was the lower scores for the conceptual polynomial problem, number two in the
104F assessment. Much of this can be attributed to the fact that one instructor did not present the polynomial material from this conceptual orientation. However, later deliberate discussions with department members on this very topic resulted in some good ideas on teaching this subject in the future. In other words, the instructor has identified a weakness and perfectly understands how to address it the next time around.

Math 205 Assessment #1 of 2

1. Timeframe for the assessment(s).
   - Fall 2011

2. Faculty involved.
   - Shuguang Li, Brian Wissman, Zorana Lazarevic, and Mitchell Anderson.

3. Student Learning Outcomes Assessed
   - ILO – Computational and Analysis
   - The following CLO’s\(^9\)
     - Demonstrate proficiency in using the standard differentiation rules
     - Utilize \(u\)-substitution when appropriate

4. Courses in which the assessment was administered.
   - Math 205

5. Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.
   - Students in these courses are typically freshmen or sophomore NS majors.
   - 124 students from six sections and four instructors participated in this assessment.

6. Details of the Assessment
   - What type of assessment was administered (Direct or Indirect), and how was the data collected?

\(^9\) See Appendix – Math 205 – 206 Course Outline
A direct assessment was employed by embedding three common problems into the final exams. This was not a major assessment effort but was intended to provide some experience to department members with the assessment processes.

How was the assessment developed?

After discussions between the four instructors Brian identified the problems and grading rubrics and presented them to Shuguang, Zorana, and Mitchell, who agreed they were at the appropriate level. The first two problems were product and quotient rule differentiation problems, respectively, and the third problem was a u-substitution anti-differentiation problem. All three problems aligned primarily with the Calculations part of the Quantitative Reasoning ILO’s, but also involved the Analysis portion as well since students cannot apply differentiation rules without understanding the role played by each function and algebraic operation, and cannot correctly apply u-substitution without understanding the chain rule.

The intent was to have the three problems embedded into the final exams for all sections. Unfortunately, poor communication and time constraints resulted in one instructor assigning the problems online via MyMathLab. For those sections about 25% of the students did not participate in the optional assessment, which may have slightly skewed the results, and in such cases questions always arise regarding the improper use of notes or other inappropriate resources. Nevertheless, the results from those sections were not inconsistent with the others, and rather than report them separately or throw them out entirely, the team decided to include them and to suggest that in the future all embedded assessments attempt to be as uniform as possible.

Note: Dr. Anderson and Dr. Wissman met in 2009 to develop embedded assessment problems for future use by the department in assessing student progress in Math 205. However, the problems they developed were intended for a more comprehensive (i.e. throughout the semester) assessment that concentrated more on key concepts rather than manipulation skills. It is suggested that these assessment problems be used by the department for a future, more comprehensive, Math 205 assessment effort.

How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)

---

10 See Appendix – Math 205 Embedded Assessment Problems and Rubrics.
11 See Appendix – Math 205 Suggested Comprehensive Assessment Problems.
Brian developed the scoring rubric, which was accepted by Shuguang, Zorana, and Mitchell. Since the scoring rubric was very straightforward, and the end of the semester was at hand, each instructor graded their own students’ work based on the rubric, and then passed the results to Brian for compilation. It was agreed that a score of 0 or 1 translated as Beginning level, 2 or 3 as Emerging, and 4 or 5 as Competent. The level of these problems did not warrant an Advanced rating for any of the three problems.

7. Results and analysis.

<table>
<thead>
<tr>
<th>Math 205</th>
<th>124 Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Product Rule</td>
</tr>
<tr>
<td>Possible Score</td>
<td>5</td>
</tr>
<tr>
<td>Average</td>
<td>4.10</td>
</tr>
<tr>
<td>QR Translation</td>
<td>Competent</td>
</tr>
</tbody>
</table>

The average scores for the differentiation problems indicated students either reached or approached competency, which is expected for these types of Math 205 problems. Approximately 76% of the students scored a 4 or 5 for the product rule, and 52% for the quotient rule. The product rule results were consistent with instructor expectations, but quotient rule results were lower than expected. Approximately 46% of the students earned a score of 4 or 5 for u-substitution. Given that u-substitution is the most difficult of the three and is the very last subject covered in 205, the results for u-substitution are consistent with expectations. It is important to note that u-substitution is again covered in the subsequent course, Math 206, and that students there generally consider it to be one of their easiest techniques to apply.

8. Closing the loop.

The results of the assessment were presented to the department for discussion in Fall 2012. Not surprisingly, they did not generate much discussion on strengths and weaknesses. However, it was readily agreed it would be highly advantageous to include within our long-range assessment plan a more comprehensive assessment project for Math 205, one that involved multiple instructors and covered more thoroughly the material covered in the course. Problems and rubrics have already been developed in

---

See Appendix – Math 205 Embedded Assessment Detailed Results
Math 205 Assessment #2

1. Timeframe for the assessment(s).
   ➢ Spring 2008 and Fall 2011

2. Faculty involved.
   ➢ Efren Ruiz and Mitchell Anderson.

3. Student Learning Outcomes Assessed
   ➢ ILO’s – Computational and Analysis
   ➢ CLO – Demonstrate proficiency in using the standard differentiation rules

4. Courses in which the assessment was administered.
   ➢ Math 205

5. Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.
   ➢ Students in these courses are typically freshmen or sophomore NS majors.
   ➢ 50 students from two sections of Efren’s class and 22 students from one section of Mitchell’s class participated in this assessment.

6. Details of the Assessment
   ➢ What type of assessment was administered (Direct or Indirect), and how was the data collected?

   These were direct assessments in which students were administered a single quiz, referred to as a “Gateway” exam\textsuperscript{13} that assessed the students’ ability to use the

\textsuperscript{13} See Appendix – Math 205 Gateway Exam.
standard differentiation rules for the usual families of functions, without simplifying. Students in Mitchell’s course were required to achieve 80% to pass, and students in Efren’s course were required to achieve 100% in order to pass. Students in Mitchell’s class were allowed to re-take a similar exam as many times as they liked, and students in Efren’s course were allowed to retake a similar exam up to 2 times per week for 8 weeks. Students were required in both sections to pass the Gateway exam in order to pass the course. The philosophy here was to encourage students to learn the derivative rules as quickly as possible, since all subsequent topics relied on correct derivatives, and students unable to regularly and efficiently apply the standard differentiation techniques to all the basic families of functions cannot find success in Math 206 and should re-take 205. Re-take exams were always different from previously given exams, but tested the same skill set.

How was the assessment developed?

Mitchell and Efren developed these assessments independent of input from the department or each other. Nevertheless, these are what would be considered standard derivative problems, covering product rule, quotient rule, and chain rule for all the standard families of functions.

How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)

No rubric was necessary. Students had to get everything correct, sans simplification, in order to receive credit. All problems were weighted equally.

7. Results and analysis.

The results for gateway exams differ considerably than most results, since they require students to demonstrate a particular level of achievement.

For Mitchell’s class, all 22 students eventually passed the Gateway exam. 14 of the 22 students passed the gateway exam on the first try. Of the remaining 8, 5 passed on the second try and one on the third. The other two required five attempts. One of these 5-attempt students went for help to the math center on numerous occasions, but seemed to have gotten confused comparing the methods used in class, particularly those for the chain rule, with the help he received there. This is not surprising since some teachers use different techniques and hence the tutors may use techniques different from each instructor. Dr. Anderson finally met with the student for a couple of hours to straighten
him out, and he passed on his next attempt. He worked hard in the class and earned a C for the course. The other 5-attempt student ended up with a D.

For Efren’s class approximately 85% of the students eventually passed the Gateway exam. Unfortunately we no longer have the data on initial success, the actual problems, or the number of subsequent attempts. 15% of the students either failed to pass the Gateway exam or gave up trying. These students did not pass the class.

8. Closing the loop.

The results of this assessment were presented to the department in Fall 2012. Not surprisingly they did not spur any discussion on strengths or weaknesses in differentiation. However, there was a lively discussion on the use of gateway exams, and what was meant by “gateway” in this case, which it was decided was a gateway to 206, and not 205. Some discussion included the possibility of utilizing these exams every time the courses were taught, and there was strong support in this direction. Overall the consensus seemed to be that there were many positive aspects to requiring such a gateway exam, and that making it policy is certainly worth considering. Discussion will continue in this regard.

Math 206 Assessment #1 of 3

1. Timeframe for the assessments.

   • Multi-year assessment conducted in Fall 2009, Spring 2010 (2 sections), Summer 2011, Fall 2011, Spring 2012, Summer 2012, and Spring 2012.

2. Faculty involved.

   • Mitchell Anderson administered the assessments. Brian Wissman and Mitchell Anderson developed the problems and rubrics.

3. Student Learning Outcomes Assessed

   • ILO’s – Computational, Analysis, Visual, and Critical Thinking Quantitative Reasoning
   • The following Math 206 CLO’s
     • Understand and apply the Fundamental Theorem of Calculus
     • Use the definition of the Definite Integral to set up problem solving models
     • Use the Definite Integral to find volumes by the washer/shell/slicing methods
Identify appropriate methods of anti-differentiation (i.e. u-sub, parts, trig, trig sub) and be able to apply the methods
Solve differential equations
Compute radius and interval of convergence for power series
Compute and manipulate McLauren Series for sin, cos, exp
Compute [at least] the first few values for a Taylor Polynomial

4. Courses in which the assessment was administered.

Math 206

5. Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.

Students in these courses are typically freshmen or sophomore NS majors, with some juniors and seniors.

125 students from seven sections participated in this assessment.

6. Details of the Assessment

What type of assessment was administered (Direct or Indirect), and how was the data collected?

A direct assessment was employed by embedding 1 – 3 common problems into each exam given throughout each semester.

How was the assessment developed?

Brian and Mitchell developed the questions and rubrics\textsuperscript{14}.

How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)

Mitchell scored the results based on the agreed upon rubrics.

7. Results and analysis.

Results for the Math 206 Multi-Year Assessment

\textsuperscript{14} See Appendix – Math 206 multi-year assessment problems and rubrics.
Note: scores are given as a percentage of the possible raw score

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>FTC</th>
<th>Def Int</th>
<th>Vol of Rev</th>
<th>u-sub</th>
<th>parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible Score</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Fall ‘09</td>
<td>12</td>
<td>.56</td>
<td>.97</td>
<td>.79</td>
<td>.62</td>
<td>.78</td>
</tr>
<tr>
<td>Spring ’10 (sec. 1)</td>
<td>14</td>
<td>.80</td>
<td>.93</td>
<td>.80</td>
<td>.66</td>
<td>.51</td>
</tr>
<tr>
<td>Spring ’10 (sec. 2)</td>
<td>15</td>
<td>.69</td>
<td>.79</td>
<td>.83</td>
<td>.75</td>
<td>.69</td>
</tr>
<tr>
<td>Summer ‘11</td>
<td>19</td>
<td>.83</td>
<td>.88</td>
<td>.97</td>
<td>.79</td>
<td>.86</td>
</tr>
<tr>
<td>Fall ‘11</td>
<td>21</td>
<td>.74</td>
<td>.90</td>
<td>.93</td>
<td>.70</td>
<td>.78</td>
</tr>
<tr>
<td>Spring ’12</td>
<td>28</td>
<td>.87</td>
<td>.84</td>
<td>.90</td>
<td>.81</td>
<td>.90</td>
</tr>
<tr>
<td>Summer ‘12</td>
<td>16</td>
<td>.65</td>
<td>.89</td>
<td>.84</td>
<td>.90</td>
<td>.85</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>.73</td>
<td>.88</td>
<td>.87</td>
<td>.75</td>
<td>.77</td>
</tr>
<tr>
<td>Institutional Goals Assessed</td>
<td>Analysis &amp; Calculations</td>
<td>Calculations</td>
<td>Analysis &amp; Visual</td>
<td>Analysis &amp; Calculations</td>
<td>Analysis &amp; Calculations</td>
<td></td>
</tr>
<tr>
<td>Quant. Reasoning Translation</td>
<td>Competent</td>
<td>Advanced</td>
<td>Advanced</td>
<td>Competent</td>
<td>Competent</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Trig sub</th>
<th>ODE</th>
<th>Interval of Convergence</th>
<th>Taylor Series</th>
<th>Taylor Series Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible Score</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Fall ‘09</td>
<td>.70</td>
<td>.40</td>
<td>.62</td>
<td>.50</td>
<td>.35</td>
</tr>
<tr>
<td>Spring ’10 (sec. 1)</td>
<td>.57</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Spring ’10 (sec. 2)</td>
<td>.67</td>
<td>.70</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Summer ‘11</td>
<td>.88</td>
<td>.73</td>
<td>.69</td>
<td>.66</td>
<td>.69</td>
</tr>
<tr>
<td>Fall ‘11</td>
<td>.63</td>
<td>.75</td>
<td>.59</td>
<td>.62</td>
<td>.34</td>
</tr>
<tr>
<td>Spring ‘12</td>
<td>.80</td>
<td>.86</td>
<td>.79</td>
<td>.70</td>
<td>.64</td>
</tr>
<tr>
<td>Summer ‘12</td>
<td>.83</td>
<td>.78</td>
<td>.83</td>
<td>.58</td>
<td>.64</td>
</tr>
<tr>
<td>Average</td>
<td>.73</td>
<td>.70</td>
<td>.70</td>
<td>.61</td>
<td>.53</td>
</tr>
<tr>
<td>Institutional Goals Assessed</td>
<td>Analysis &amp; Calculations</td>
<td>Calculations</td>
<td>Analysis &amp; Calculations</td>
<td>Analysis &amp; Calculations</td>
<td>Analysis &amp; Calculations</td>
</tr>
<tr>
<td>QR Translation</td>
<td>Competent</td>
<td>Competent</td>
<td>Competent</td>
<td>Competent</td>
<td>Emerging</td>
</tr>
</tbody>
</table>

15 NA indicates missing data.
One of the advantages of a multi-year assessment is to be able to look for trends from year to year. While very few of the ten problems indicate a monotonic increase, most show a general trend towards improvement. Fluctuations in the preparedness and quality of students from semester to semester undoubtedly accounts for some of the lack of monotonicity.

Results indicate that for all but one question the average for the students over the six semesters reached a level of Competent or Advanced. More notable is that for the first six questions at least two of the last four semesters achieved advanced ratings, and problems 7 and 8 missed this mark by only 1 or 2 percentage points. This represents a marked improvement over the first three semesters for most of the six questions. Infinite series, represented by the last three questions, clearly remains the most difficult for the students. Dr. Anderson made adjustments over the years in an effort to ensure that students had more time to absorb the material by moving some of his deadlines, so that students could concentrate solely on infinite series when they encountered the subject.

Weaknesses:

i. FTC – the most common misunderstanding for this type of problem is students’ misinterpretation of the integral for negative values. That is to say they compute the negative area incorrectly, computing the area of rectangles or triangles that have an incorrect base length or they use the area that is actually below the negative portion of the graph, drawing the base at the lowest height. It seems they attempt to visually
compute an area, which they know is to be negative, but they ignore the definition of the definite integral as a limit of approximating sums, each rectangle resulting from the height at an x value within the interval where f is negative. So, while they get a negative value, it is computed using the wrong x-interval, where f is actually positive, or they compute the area using the correct x-interval but the wrong heights. The second most common mistake is to break up triangles, the area of which can obviously easily be computed, into many small rectangles, and guessing the values of the many non smaller triangular shapes at the edges.

ii. Definite Integral – this problem is very straightforward. Common mistakes are not antidifferentiating at all, or doing it incorrectly. Another less common mistake is to compute \( F(a) - F(b) \) instead of \( F(b) - F(a) \).

iii. Volumes of Revolution – the scores here are very high in general. The most common mistake is to use the shell method, obviously confusing the two methods.

iv. U-sub – the most common mistakes result from not being organized. In many instances it is difficult to figure out what substitution they make for u, and then in still other cases they are not explicit about changing the dx integral into a du integral, with the important steps missing. Another common mistake is mixing x’s and u’s into the same integral. The most important mistake seems to be that students do not understand that once the substitution is made, if the resultant du integral is not easy, then the process was probably not worthwhile. A sad mistake is when students do not recognize simple u-substitution problems as being such, which indicates they do not truly understand the chain rule or the structure of its results.

v. Parts – the most common mistake with parts is not recognizing that the problem is a parts problem. This problem is a tabular (i.e. repeated) parts problem involving a power of x and a simple sin, cos, or exp. The most common problems here are not correctly anti-differentiating correctly sin 3x, either getting sin 3x/3, or 3cos3x, or missing the negative sign.

vi. Trig sub – this appears to be one of the more difficult techniques for students. First, they have difficulty recognizing it as a trig sub problem. The only explanation for a problem that is so clearly trig sub is that they are confusing the different methods and do not have a good understanding of any of them. One of the more common mistakes students make, once they attempt to apply a trig sub is to pick the wrong trig function, again a silly mistake. Some of the better students even do this. The most obvious weakness associated with this type of problem is lack of organization. There are a lot of steps to this problem and if students are not neat and organized it is easy to lose ones step along the way. Back substitution is the final weakness.

vii. ODE – this problem is very straightforward and the scores should be higher. Students seem to understand that they need to take the appropriate number of derivatives and then plug them all in to see if “it works”, but they are weak in two areas. Some cannot take multiple derivatives. If they use the chain rule the first time it is hard for
some to decide they need to use the product and the chain rule the second time, etc. Then, they have a hard time “showing” that it works. They do not finish the problem properly, stating that the left side of the equation does not equal the right.

viii. Interval of Convergence – applying the ratio test to ratios that involve a variable seems to be the biggest weakness, although by this point it is not uncommon for students to have completely run out of time for these last three problems, not in terms of the test but in terms of being able to study, so they flunk miserably on this part of the exam. Common mistakes are students getting the radius of convergence and not checking the endpoints correctly. It seems they plug everything in correctly but then gloss over the necessary details or analysis of the resulting infinite series.

ix. Taylor Series – there are two primary mistakes that students make on this particularly simple problem. First, they do not recall the McClaurin series, and second they incorrectly evaluate at $x^2$, which is a precalculus type of skill. Again, it is unfortunate but some students are pretty worn out by this stage of the course. Of course, some students find this problem extremely easy.

x. Taylor Series Coefficients – the first part of this problem is identical in terms of the level of difficulty as the one directly above. A common mistake is differentiating incorrectly. Another is not writing down a series at all. Simple algebra mistakes are common as well. It is interesting to note that this problem probably has the highest standard deviation for its scores. There are a lot of full credit scores for this problem, and unfortunately a lot with a zero. This indicates that if the students are caught up with their work they can certainly master the main concepts here. It is the ones who are behind in the class, and probably in others as well, that struggle to catch up at the end and their infinite series studies suffer, particularly the very last part that we cover, which is the type of problem covered in this problem.

Analysis of this assessment through the Institutional Goals lens

The concepts covered in Math 206 are at the high end of Quantitative Reasoning related concepts, particularly in terms of critical thinking and analytic skills. The problems chosen for this assessment, some of which are the more basic questions encountered in Math 206, still reflect this idea. The Fundamental Theorem of Calculus problem requires students to understand the implications of FTC, and then to apply them in a real world setting, performing multi-level computations and analysis. The volume of revolution problem requires students to visualize in three dimensions and to be able to extract information from that visualization. It also requires students to understand the concept of integration from a “one piece at a time” perspective and to extend that understanding to the implications of the definition of the definite integral as the limit of a sum, thereby allowing the transformation of the sum of individual pieces to a Definite Integral that can easily be computed either by hand or with the use of technology. The integration techniques problems require students to identify and analyze differentiation from the
results end, and to be able to move between differentiation and anti-differentiation within the same problem, combined with faultless precalculus skills. The infinite series questions push students’ analytic and computational skills to their limit, no pun intended. It is extremely pleasing to students and instructors alike to reflect on the students’ abilities to tackle problems of such advanced mathematical sophistication, given the fact that for many it was less than six months prior that they had even learned the simple skill of differentiation. Students achieving competent scores for such Math 206 problems are certainly at the advanced levels intended by the Quantitative Reasoning institutional goals.

8. Closing the loop.

As this assessment progressed Dr. Anderson made adjustments in his teaching, most notably allowing more instructional time for those topics the assessment results indicated students were struggling with, and pointing more frequently to the types of misunderstandings of former students, as noted in the weaknesses above, while still avoiding at all times a possible tendency to teach to the test. A discussion was held with the department on the results in Fall 2012 and a short discussion ensued as to what the faculty considered important learning outcomes for Math 206. It was also suggested that we could use a more comprehensive assessment across all sections of Math 206. This is one of the items included in our revised long range assessment plan.

**Math 206 Assessment #2 of 3**

1. Timeframe for the assessment.
   - Spring 2012.

2. Faculty involved.
   - Brian Wissman and Mitchell Anderson. (Shuguang Li was involved initially, but did not complete the assessment.)

3. Student Learning Outcomes Assessed
   - ILO’s – Calculations, Analysis, and Critical Thinking (Quantitative Reasoning)
   - The following Math 206 CLO’s
     - Identify appropriate methods of anti-differentiation (i.e. u-sub, parts, trig, trig sub) and be able to apply the methods
     - Compute radius and interval of convergence for power series
Compute and manipulate McLauren Series for sin, cos, exp
Compute [at least] the first few values for a Taylor Polynomial

4. Courses in which the assessment was administered.

- Math 206

5. Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.

- Students in these courses are typically freshmen or sophomore NS majors, with some juniors and seniors.
- 65 students from three sections participated in this assessment, 26 from Mitchell’s single section and the remainder from Brian’s two sections.

6. Details of the Assessment

- What type of assessment was administered (Direct or Indirect), and how was the data collected?
  
  This was a direct assessment in which 6 problems were embedded into two exams, 3 in the techniques of integration exam and 3 in the infinite series exam. (Note: the results for Mitchell’s exams are also included in his multi-year assessment, with the difference being that Brian’s results are included here as well, and Mitchell’s first 3 multi-year problems as well as the ODE problem are omitted here.)

- How was the assessment developed?
  
  Brian and Mitchell developed the problems long ago. These problems were either taken directly from those used in the multi-semester assessment #1 above, or were very similar, with essentially identical rubrics.

- How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)
  
  Brian and Mitchell developed the rubrics at the same time they identified the problems. They scored their results separately using the common rubric and combined the results.
7. Results and analysis.

<table>
<thead>
<tr>
<th>Possible Score</th>
<th>u-sub</th>
<th>Parts</th>
<th>Trig Sub</th>
<th>Interval of Convergence</th>
<th>Taylor Series</th>
<th>Taylor Series Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sp 2012</td>
<td>.77</td>
<td>.84</td>
<td>.48</td>
<td>.59</td>
<td>.47</td>
<td>.51</td>
</tr>
</tbody>
</table>

The results here indicate that students are struggling with Trig Sub and most aspects of infinite series.

8. Closing the loop.

The results of this assessment were presented at the same time as those for Math 206 assessment number one above. It was noted that one of the reasons the scores were a bit lower here was that some of the students did not even attempt some of the more difficult problems, such as trig sub, because the exam was a bit long. A discussion ensued as to how to ensure that if you embedded problems for assessment within an exam students would actually attempt it, resulting in a valid assessment of their capabilities for each problem. In the future this will be dealt with. Mitchell noted that he did not experience the same problem, making sure that time was not a critical issue for his exams.

**Math 206 Assessment #3 of 3**

1. Timeframe for the assessment.
   - Spring 2012.

2. Faculty involved.
   - Brian Wissman and Mitchell Anderson.

3. Student Learning Outcomes Assessed
   - ILO’s – Computational, Analysis, Visual, and Critical Thinking (Quantitative Reasoning), and Organization and Structure (Communication)
The following Math 206 CLO’s

- Understand and apply the Fundamental Theorem of Calculus
- Use the definition of the Definite Integral to set up problem solving models
- Understand and apply Integration as a process

The following CLO’s from the lab portion of Math 206

- Determine when it is more appropriate to use technology for Math 206 level problems
- Use technology to compute various approximations to a definite integral (e.g. Left(n), Right(n), and the best available approximation)
- Use technology to compute a sequence of approximations to an applied definite integral problem, and make reasonable conclusions regarding convergence to the answer
- Apply Euler’s method and technology to create a sequence of approximations

4. Courses in which the assessment was administered.

- Math 206

5. Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.

- Students in these courses are typically freshmen or sophomore NS majors, with some juniors and seniors.
- Approximately 27 students from three sections participated in this assessment, 15 from Mitchell’s single section (about half his class) and the remainder from Brian’s two sections.

6. Details of the Assessment

- What type of assessment was administered (Direct or Indirect), and how was the data collected?

This was a direct assessment consisting of a written group project. Mitchell has given two group projects for numerous years in his Math 206 course. Students typically work in groups of three, assigned by the instructor, and hand in a single written report
for which they each receive the same grade. Mitchell has noted a marked decline in the quality of writing for his projects, which prompted this assessment.

Students in Dr. Anderson’s class were provided, in addition to the group project, a document entitled Group Project Template\textsuperscript{16} to assist them in understanding how the report should be written. Students were given a few weeks to work on the projects and each group turned in a single report, usually 2 – 3 pages in length. Students in Dr. Wissman’s class worked on their projects individually.

- How was the assessment developed?

Brian and Shuguang developed the \textit{Lead Balloon Analysis}\textsuperscript{17} for their project and Mitchell used his usual \textit{River Skipper}\textsuperscript{18} group project. Brian and Mitchell met and discussed in detail the intent of the projects.

The purpose of this assessment was multifold.

- To provide experience working collaboratively on larger endeavors.

- To further explore and understand integration as a process.

- To improve written communication skills.

- How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)

A cross section of five of the total ten reports was selected from Mitchell’s class, and a cross section of four reports was selected from Brian’s class. Shuguang was not involved in this portion of the assessment. Brian and Mitchell decided to limit the number of reports to a cross section of each class to reduce the amount of work. However, they attempted to choose a representative cross section for both projects.

Brian and Mitchell met and after considerable discussion developed a scoring rubric prior to scoring the projects.\textsuperscript{19} They agreed that these projects assessed both Quantitative Reasoning ILO’s, including a more than usual level of Critical Thinking, and Communication ILO’s. From the Quantitative Reasoning ILO’s they identified both Analysis and Calculations. From the Communication ILO’s they identified Organization and Structure, and Line of Reasoning. When they attempted to

\textsuperscript{16} See Appendix – Math 206 River Skipper Template
\textsuperscript{17} See Appendix – Math 206 Lead Balloon Analysis
\textsuperscript{18} See Appendix – Math 206 River Skipper
\textsuperscript{19} See Appendix – Math 206 Writing Project Rubric
distinguish which student learning outcomes to assess within each institutional goal it became clear that the Line of Reasoning outcome within Communications was directly tied to the Analysis outcome within Quantitative Reasoning. Students needed to identify the appropriate strategies to mathematically solve the problem, which falls within Analysis, and they needed to communicate them well enough for an outsider to understand, which falls within Line of Reasoning. Thus, they decided to first score Communication based on C1 – Professional Layout, C2 – Level of Prose, C3 – Communication of the Solution, and Quantitative Reasoning based on Q1 – Strategy, and Q2 – Results. They then averaged C1 and C2 (Organization and Structure), C3 and Q1 (Critical Thinking, Analysis, and Line of Reasoning).

After scoring each project individually they met again to compare results and discuss any minor discrepancies.

7. Results and analysis.

The philosophy guiding Dr. Anderson’s projects is that in today’s work environment fellow employees or colleagues often collaborate on large projects and must then present their results. Although Dr. Anderson has received many such reports over the years and has gone to great length to discuss in class the type of reports he expects, the reports he received this semester were unacceptable. He returned them to the students with instructions to re-do them. The poor quality of these “first drafts” prompted the assessment and prompted him to make more explicit his expectations by revising his Group Project Requirements and providing the students with the new version titled Group Project Template. The final drafts were much improved. In the table below A1, A2, etc. refer to group projects from Dr. Anderson’s class and W1, W2, etc. to those in Dr. Wissman’s class.
Math 206 Writing Project Results
Possible scores 1 - 4

<table>
<thead>
<tr>
<th>Individual Reports</th>
<th>CI/C2</th>
<th>C3/Q1</th>
<th>Q2</th>
<th>Avg. Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>4</td>
<td>3.75</td>
<td>4</td>
<td>Advanced</td>
</tr>
<tr>
<td>A2</td>
<td>3</td>
<td>2.5</td>
<td>2.5</td>
<td>Approaching Competent</td>
</tr>
<tr>
<td>A3</td>
<td>3.5</td>
<td>2.25</td>
<td>2.5</td>
<td>Approaching Competent</td>
</tr>
<tr>
<td>A4</td>
<td>2.75</td>
<td>2.25</td>
<td>3</td>
<td>Approaching Competent</td>
</tr>
<tr>
<td>A5</td>
<td>2.75</td>
<td>2</td>
<td>1.5</td>
<td>Emerging</td>
</tr>
<tr>
<td>W1</td>
<td>1.5</td>
<td>1.75</td>
<td>2</td>
<td>Emerging</td>
</tr>
<tr>
<td>W2</td>
<td>3</td>
<td>3.25</td>
<td>3</td>
<td>Competent</td>
</tr>
<tr>
<td>W3</td>
<td>3</td>
<td>2.25</td>
<td>2</td>
<td>Emerging</td>
</tr>
<tr>
<td>W4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Beginning</td>
</tr>
</tbody>
</table>

Institutional Goals Assessed
Organization & Structure  Reasoning & Analysis & Critical Thinking  Computations

These writing projects are considerably more difficult than other work performed in the class, both in terms of the deeper level of critical thinking and understanding that is required and in terms of the need to communicate the solution in a manner conducive to the reader being able to understand the problem, the strategies for solving the problem, the steps utilized, and the results. For many students Math 206 is their last math class, and our last opportunity to contribute to their education and prepare them for the real world. As such our goal is for most students to demonstrate capabilities at the level of competent or above. While not every student reached this level, more than half reached an average level of Approaching Competency or better.

Weaknesses: The strategy for Mitchell’s project, a non-standard integration problem, requires taking numerical approximations to time and distance traveled and then taking better and better approximations to simulate the convergence of a limit. In many instances students only utilize the first part of this strategy and omit the important second part. In other cases they state that “by the distance formula” something must be true, but it is not always clear if they are referring to the $d = rt$ formula or the distance between two points, both of which need to be used in different places for this problem. This particular weakness is a reflection of deficient communication skills, which show up
throughout the students’ writings. In other cases the entire strategy is wrong, in essence they missed the proverbial boat entirely, in which case their written explanations must necessarily be exceedingly poor as well. Two of the most common strategy mistakes are to try to find average velocity by averaging a bunch of velocities, and the second is to use horizontal distance in place of a linear approximation to distance traveled over a short x-interval. One of the more common communication weaknesses is to include whole paragraphs of irrelevant material, usually explaining how to find the distance the boat traveled. While this may be an interesting process, since they usually break up the river into small pieces and essentially apply the basis for the arc length theorem, which they could have alternatively used in any case, they never use this piece of information elsewhere in the problem.

Note: It should be noted that it is possible to answer this problem by setting up a definite integral, provided the velocity function used does not include a zero. Over the years only one group has ever achieved this, but it appeared at the time that it may have been by trial and error, or they simply tried to figure out a distance formula divided by velocity. The student who achieved this was pretty talented, so he may have understood what he did, but the justification was not very thorough.

In some cases the organization and structure of the writing was very poor. When the organization was adequate students still needed to overcome the challenge that it is very difficult to say just what is true and at just the right level of detail, and most of the students’ papers reflected this difficulty at some level.

8. Closing the loop.

The results of this assessment were presented and discussed in a department meeting. The topic of student writing capabilities and weaknesses were discuss. As a result of this assessment Dr. Anderson implemented four short writing assignments into subsequent Math 206\textsuperscript{20} classes and one in his Math 205\textsuperscript{21}. He also developed a revised writing template for his group project that was more explicit and better guided the students in presenting a logically flowing and relevant report. The four short writing assignments used a very similar template and were intended to better prepare his students for the more detailed projects. He noted that the student writing improved notably from the first to the fourth, and these assignments helped students with their group projects as well, as the template and writing style became more familiar as the students progressed through the course.

\textsuperscript{20} See Appendix – Math 206 Writing Assignments 1 – 4, Overview, and Rubric
\textsuperscript{21} See Appendix – Math 205 Writing Assignment and Overview
Math 311 Assessment

1. Timeframe for the assessment.
   - Spring 2010 and Spring 2012.

2. Faculty involved.
   - Mitchell and Brian developed the problems and rubrics. Mitchell conducted the assessments.

3. Student Learning Outcomes Assessed
   - ILO’s – Calculations, Analysis, Visualization and Critical Thinking (Quantitative Reasoning)
   - The following Math 311 CLO’s
     - Demonstrate an understanding of the structure of solutions to linear systems of equations. (eg. Given a single solution to $AX = B$, find and use the solutions to the homogeneous equation $AX = 0$ to find all solutions to $AX = B$.)
     - Apply the subspace theorem to show a subset under the inherited operations forms a vector space.
     - Show that a set of vectors in a vector space is linearly independent/dependent.
     - Find a basis for a [non-standard] vector space.
     - Compute the eigenpairs for a matrix.
   - The following Math Degree PLO’s
     - Demonstrate mastery of the core material found in single and multi-variable Calculus and Linear Algebra.
     - Demonstrate mastery of the core concepts in Abstract Algebra and Real Analysis.
     - Identify, compare, and contrast the fundamental concepts within and across the major areas of mathematics, with particular emphasis on Linear Algebra, Abstract Algebra, and Real Analysis.
     - Use a variety of theorem-proving techniques to prove mathematical results.
     - Demonstrate the abilities to read and articulate mathematics verbally and in writing.

---

22 See Appendix – Math 311 Problems and Rubrics.
4. Courses in which the assessment was administered.

- Math 311

5. Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.

- Students in these courses are typically Sophomore or Junior Math or Computer Science majors.

- The Spring 2010 assessment involved 16 students and the Spring 2012 assessment involved between 7 and 13 students (this class had a very high drop rate, with surprisingly many students unprepared for the level of work).

6. Details of the Assessment

- What type of assessment was administered (Direct or Indirect), and how was the data collected?

This was a direct assessment in which 1 and 3 problems were embedded into three exams, for a total of 5 questions each semester. The problems used within the more comprehensive set of problems developed by Mitchell and Brian were System of Equations #2, Vector Spaces #2, Linear Independence #2, Basis and Span #1, and Eigenvalues/Eigenvectors #1.

These five problems are representative of the combination of skills and concepts students should be exposed to and learn in Math 311. Showing a set of vectors is linearly independent and finding eigenpairs are primarily computational problems that can be successfully completed without a deep understanding of the underlying concepts. On the other hand, finding all solutions to a non-homogeneous system, given only one solution and a row-equivalent representation of the coefficient matrix, requires students to understand the fundamental structure of solutions to linear systems as well as being able to perform rudimentary algebra. Similarly, proving a subset of vectors forms a vector space is for some students their first foray into theorem proving, and the specific problem used requires familiarity with the concepts of functions that can only be gained through practice from earlier classes. Finally, finding a basis, even for the simplest set of non standard vectors, requires a deep understanding of vectors, linear independence, and span, but no computations.
How was the assessment developed?
Brian and Mitchell developed the problems and rubrics.

How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)
See scoring rubric. Mitchell scored the problems himself, based on the agreed upon rubric.

7. Results and analysis.
This assessment covered student learning outcomes within both Math Dept. Program Goals and Institutional Goals. As indicated by the department’s Course Alignment Matrix, Linear Algebra crosses all program SLO’s. Accordingly, each of the problems utilized within this assessment required students to demonstrate their understanding of at least one standard Linear Algebra Concept. Not surprisingly, this assessment also assessed learning outcomes found within the Quantitative Reasoning institutional goal, including critical thinking, analysis, and computation. The specific program and institutional learning outcomes addressed by each problem are included in the results table below. Critical thinking, while not listed in the institutional goals, is included in each and is not listed for convenience.

Results for the Math 311 Assessment

<table>
<thead>
<tr>
<th></th>
<th>Sys Eqns</th>
<th>Vec Spaces</th>
<th>Lin Ind</th>
<th>Bases/Span</th>
<th>Eigen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible Score</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Spring 2010</td>
<td>1.89</td>
<td>3.88</td>
<td>2.81</td>
<td>2.39</td>
<td>3.25</td>
</tr>
<tr>
<td>Spring 2012</td>
<td>2.15</td>
<td>2.89</td>
<td>2.33</td>
<td>1.11</td>
<td>3.14</td>
</tr>
<tr>
<td>Avg %</td>
<td>.67</td>
<td>.85</td>
<td>.86</td>
<td>.58</td>
<td>.80</td>
</tr>
<tr>
<td>Program SLO’s</td>
<td>1 &amp; 3</td>
<td>1-5</td>
<td>1 &amp; 3</td>
<td>1 &amp; 3</td>
<td>1 &amp; 3</td>
</tr>
<tr>
<td>Institutional Goals Assessed</td>
<td>Analysis &amp; Calculations</td>
<td>Analysis</td>
<td>Analysis &amp; Calculations</td>
<td>Analysis</td>
<td>Analysis &amp; Calculations</td>
</tr>
<tr>
<td>Translation</td>
<td>Competent</td>
<td>Advanced</td>
<td>Advanced</td>
<td>Approaching Competence</td>
<td>Advanced</td>
</tr>
</tbody>
</table>

Again, with a multi-year assessment it would be nice to see improvement from year to year. However, in hindsight the Spring 2012 semester was perhaps not the best in which to conduct an assessment. It began with 16 students, ended with 13 registered but only 7 taking the final exam. It was the worse class in terms of sporadic attendance that
Mitchell has experienced in his long career. That said, a comparison of scores in such a small sample is perhaps not all that useful. On the other hand, the scores from Spring 2010 indicate the students had a very good grasp of the material and except for the first problem consistently reached levels that we could describe as competent or advanced. It appears that the students in that class improved dramatically after their first exam, which Mitchell has found typical for Math 311.

Weaknesses: As long as the students have a good understanding of the basic concepts, each of the problems used is very straightforward.

Systems of Equations – The system of equations problem requires students to identify the solution space to a homogeneous system, given by a reduced 3 x 3 matrix, and to combine those results with one solution to an associated non-homogeneous system. Most of the mistakes on this problem result from not understanding the fundamental structure of such systems, not from computational mistakes.

Vector Spaces – This problem requires students to show that the subset of P₃ with roots at 1 and 2 forms a subspace. The biggest weakness for this problem is not knowing that in order to check closure of addition you need to check whether the sum of two such functions still has roots at 1 and 2. This is a weakness in being able to read mathematics. Students also frequently simply write down necessary results with no work shown, in essence hard wiring the work to indicate it has closure because it has closure. They fail to see, for example, the significance of the difference between addition in P₃ and addition in the real numbers, each playing a role in this problem.

Linear Independence – This problem is highly computational and really only requires one to take the determinant of a 3 x 3 real matrix and check to see if the result is zero or not. The only obvious weakness for this problem, apart from not being careful with the computations, is not realizing that the determinant is all that is required.

Bases and Span – This problem requires no computation; it only requires students to list a basis for a subspace of M₃. For a mathematician this is a trivial problem, and as long as the student understands a couple of key concepts, that is still the case. However, students tend to make very serious mistakes on this problem. The biggest mistake, and the most common, is to give as their answer a single or set of vectors that do not even lie in M₃. This indicates a severe misunderstanding of the underlying concepts. Mitchell has noted to the students that this is the biggest mistake past students have made, and provides numerous examples. Nevertheless, many students continue to make this critical error.
Eigenvalues and Eigenvectors – This problem simply requires students to identify eigenpairs for a 2 x 2 matrix. It is highly computational, but has numerous steps. There does not appear to be any serious misunderstandings associated with this problem. That is not to say that students do not find ways to make critical errors. One aspect that is worrisome is that students do not seem to check their answers, which seems to indicate they do not really understand the concept of Eigenvectors and Eigenvalues.

8. Closing the loop.

These results were presented at a Fall 2012 department meeting and a short discussion ensued. Due to time constraints and the limited nature of the discussion, alternatively faculty members were encouraged to pay particular attention to the weaknesses that were identified and noted in the assessment report and to adjust their teaching accordingly. However, until there is increased participation in assessment, with increased discussion of the results, it is unlikely that this assessment will benefit their teaching. It is the hope of those that have been more fully involved that once the department begins following the assessment plan, with two assessments presented each year, ample time will be allotted for discussion and faculty members will become more engaged in the continuous improvement process.

Math 431 Assessment

1. Timeframe for the assessment.

   Fall 2010 - Spring 2011.

2. Faculty involved.

   - Mitchell Anderson developed the problems and was the instructor. Mitchell, Brian Wissman, Efren Ruiz, and Roberto Pelayo developed the rubrics. Mitchell conducted the assessments, and the team did the scoring.

3. Student Learning Outcomes Assessed

   - ILO’s – Calculations, Analysis, Visualization and Critical Thinking (Quantitative Reasoning)

   - The following Math 431 CLO’s

---

23 See Appendix – Math 431-432 Assessment Problems and Rubrics. Also see Appendix – Math 431-432 Full Problem Set for a complete list of the problems presented in the course.

24 See Appendix – Math 431-432 Course Outline
- Demonstrate an understanding of the basic theorems and their implications regarding the topology of the line (and Euclidean n-space if they take 432), sequences and subsequences, compactness, denseness, convergence, continuity, differentiability, the definite integral, cardinality, and metric spaces.
- Utilize a variety of standard theorem proving techniques to construct valid proofs, presented in a logically correct order (e.g. defining variables before using them).
- Construct counter-examples to false statements, and when feasible conjecture hypothesis that would make the statements true.

➢ The following Math Degree PLO’s

- Demonstrate mastery of the core material found in single and multi-variable Calculus and Linear Algebra.
- Demonstrate mastery of the core concepts in Abstract Algebra and Real Analysis.
- Identify, compare, and contrast the fundamental concepts within and across the major areas of mathematics, with particular emphasis on Linear Algebra, Abstract Algebra, and Real Analysis.
- Use a variety of theorem-proving techniques to prove mathematical results.
- Demonstrate the abilities to read and articulate mathematics verbally and in writing.

4. Courses in which the assessment was administered.

➢ Math 431-432

5. Audience (i.e. Math majors, NS majors, non-science majors), levels (i.e., freshmen/soph or junior/senior), and number of participants.

➢ Students in these courses are typically Junior or Senior Math majors.

➢ The Fall 2010 class has 16 students and the Spring 2011 class had 14 students.

6. Details of the Assessment

➢ What type of assessment was administered (Direct or Indirect), and how was the data collected?
This was a direct assessment in which Mitchell gathered a portfolio of 21 proofs presented in class. In this course students were provided definitions and statements that may or may not have been true. Students were required, without the benefit of lecture, discussion, or textbook to identify if a statement was true or false, and in the case it was false to provide a counterexample and perhaps a change of hypothesis to make it true, and in the case it was true to provide a proof. Students then orally presented their results in class. For this academic year Dr. Anderson asked some students to provide him with an electronic copy of their presentations once the class felt they were valid. Dr. Anderson compiled those results into a class portfolio\textsuperscript{25}, including 21 proofs, removed the names, and the four department members scored them based on their common rubrics. They then assembled to discuss their scores and any discrepancies.

Since most of the work had been validated by the rest of the class, most of the portfolio is free from error and hence does not offer the range of results found in other assessments. Nevertheless, it does demonstrate that ILO’s, PLO’s and CLO’s are being met at an advanced level. The 21 proofs ranged from theorems involving the existence of limit points and limits of sequences, to proving that continuous functions over a compact set are uniformly continuous, Cauchy sequences converge, every point in the Cantor set is a limit point of the Cantor set, which is also of measure zero, and integrability existence theorems.

Note: Math 431 – 432 is one of the most difficult course sequences offered by the department, and when offered in this format provides students an unparalleled opportunity for intellectual growth. Since all of the proofs were “accepted” through a peer reviewed presentation process it was expected that most of the scores would be high. This type of assessment appears to be appropriate for this type of class if one wants to assess the level of achievement possible by the class as a whole, particularly in terms of providing students the necessary preparation for graduate school. That being said, in Mitchell’s opinion this particular class did not stand out as exceptional in terms of the quantity and quality of the proofs presented.

How was it analyzed? (e.g. What type of scoring rubric was developed, who developed it, and who did the scoring?)

Mitchell, Bob, Brian, and Efren developed the rubrics. Mitchell removed the names from each proof, made copies, and passed them along to the team. Each team member then scored a sample of five of the proofs individually, based on the rubric, and then the team met to discuss the results.

\textsuperscript{25} See Appendix – Math 431 Portfolio
7. Results and analysis.
Overall it was agreed that the level of work demonstrated that the class offered the appropriate level of work in order to meet the PLO numbers 2, 4, and 5. Outcome 2 requires mastery of the core concepts of Abstract Algebra and Analysis. Outcome 3 requires students to use a variety of theorem proving techniques. Outcome 4 requires students to demonstrate the abilities to read and articulate mathematics verbally and in writing. The five theorems the team evaluated included the intermediate value theorem, the convergence of Cauchy sequences, countable number set have measure zero, and the property of bounded variation for non-decreasing functions. Some of the comments the team made dealt with questioning whether or not standard notation was followed for the students (it was), in particular when defining sequences, requirements to include every detail within a proof, and whether or not students were covering every case when more than one case held (e.g. the set in question was finite or infinite).

The team did not find any deficiencies it felt needed addressing, noting that the work indicates a proper level for preparing students for graduate level mathematics. It did however suggest that it would be nice to conduct a more thorough assessment in the future, one that tracked individual student performance and improvement from the beginning of the class to the end, to show value added. The department’s assessment plan has a spot designated for something involving outcomes 2 – 5, which might address this well.

8. Closing the loop.

The results were discussed at a full department meeting. The department echoed the views of the assessment team and agreed that a future more comprehensive assessment might provide valuable information on a more refined level.