PHYS170: General Physics I: Mechanics

Equations & Constants

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Constants

- Speed of light (in vacuum): $c = 2.998 \times 10^8\,\mathrm{m\,s^{-1}}$
- Gravitational constant: $G = 6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
- Gravitational acceleration at Earth's surface: $g = 9.807\,\mathrm{m\,s^{-2}}$
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Object (symb.)	Mass (kg)	Radius (m)
Earth ($\oplus$)	5.972×10^{24}	6.378×10^6
Moon ($\leftmoon$)	7.394×10^{22}	1.738×10^6
Sun ($\odot$)	1.989×10^{30}	6.957×10^8
Electron (e^-)	9.109×10^{-31}	$< 10^{-16}$
Proton (p^+)	1.673×10^{-27}	$\approx 10^{-15}$
Neutron (n^0)	1.675×10^{-27}	$\approx 10^{-15}$

- Earth-Sun distance (astronomical unit): $1\,\mathrm{AU} = 1.496 \times 10^{11}\,\mathrm{m}$
- Earth-Moon distance: $3.844 \times 10^6\,\mathrm{m}$
- Pressure: $1\,\mathrm{Pa}$ (pascal) $= 1\,\mathrm{N\,m^{-2}}$
  $1\,\mathrm{Pa} = 9.869 \times 10^{-6}\,\mathrm{atm}$ (atmosphere)
- Planck constant: $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$
- Boltzmann constant: $k_\mathrm{B} = 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
- Permittivity of vacuum: $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
- Fundamental quantum of charge: $e = 1.602 \times 10^{-19}\,\mathrm{C}$ (coulomb)
- Coulomb constant: $k_e = \dfrac{1}{4\pi\varepsilon_0} = 8.988 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$

Equations

- Density: $\rho = \dfrac{M}{V}$
- Temperature: $T_{(^\circ\mathrm{C})} = T_\mathrm{K} - 273$ and $T_{(^\circ\mathrm{C})} = \dfrac{5}{9}\left(T_{(^\circ\mathrm{F})} - 32\right)$
- Magnitude of static friction: $f_\mathrm{s} \le \mu_\mathrm{s} N$
- Magnitude of kinetic friction: $f_\mathrm{k} = \mu_\mathrm{k} N$
- Gravitational force: $\vec{\mathbf{F}}_{12}(r) = \dfrac{G m_1 m_2}{r^2}\,\hat{\mathbf{r}}_{12}$
- Electromagnetic or Coulomb force: $\vec{\mathbf{F}}_{12}(r) = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}\,\hat{\mathbf{r}}_{12}$, where $q_\#$ are charges.
- Drag force: $\vec{\mathbf{F}}_\mathrm{D} = -\dfrac{1}{2} C\rho A v^2\,\hat{\mathbf{v}}$, where C is drag coefficient, A is area, and ρ is density of fluid.
- Hooke's law: $\vec{\mathbf{F}} = -k(\vec{\mathbf{r}} - \vec{\mathbf{r}}_\mathrm{eq})$, where k is spring constant and $\vec{\mathbf{r}}_\mathrm{eq}$ is its equilibrium position.
- Pressure: $P = \dfrac{F}{A}$, where F is force and A is area.
- Ideal gas law (gas pressure): $PV = N\,k_\mathrm{B}\,T$, where N is number of particles.
- Kinematic equations (constant $\vec{\mathbf{a}}$):[†]
  $\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$
  $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \dfrac{1}{2}\vec{\mathbf{a}}t^2$
  $v^2 = v_0^2 + 2a(r - r_0)$ (by component)
- Quadratic solution for $ax^2 + bx + c = 0$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Free-fall with resistive force $\vec{\mathbf{f}}_\mathrm{R} = -b\vec{\mathbf{v}}$, where b is constant and positive: $\vec{\mathbf{v}}(t) = \dfrac{m\vec{\mathbf{g}}}{b}\left(1 - e^{-bt/m}\right)$
- Impulse: $\vec{\mathbf{J}} = \displaystyle\int_{t_0}^{t_\mathrm{f}} \vec{\mathbf{F}}(t)dt = \vec{\mathbf{p}}_\mathrm{f} - \vec{\mathbf{p}}_0$
- Angular momentum: $\vec{\mathbf{L}} = m\vec{\mathbf{v}} \times \vec{\mathbf{r}}$ and $\dfrac{d\vec{\mathbf{L}}}{dt} = \displaystyle\sum_{j=1}^{\mathcal{N}} \vec{\boldsymbol{\tau}}_j$
- Angular velocity ($\vec{\boldsymbol{\omega}}$): $\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$, also called angular frequency: $\omega = \dfrac{2\pi}{T}$
- Angular acceleration ($\vec{\boldsymbol{\alpha}}$): $\vec{\mathbf{a}} = \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}$
- Torque: $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
- Centripetal acceleration: $\vec{\mathbf{a}}_\mathrm{cent} = -\dfrac{v^2}{r}\,\hat{\mathbf{r}} = -\omega^2\vec{\mathbf{r}}$
- Center of mass of system of particles: $\vec{\mathbf{r}}_\mathrm{CM} = \dfrac{1}{M}\displaystyle\sum_{j=1}^{\mathcal{N}} m_j \vec{\mathbf{r}}_j$
- Moment of inertia for system of particles: $I = \displaystyle\sum_{j=1}^{\mathcal{N}} m_j r_j^2$. Moments for extended objects:
  - Hoop about cylinder axis (C.A.): $I = MR^2$
  - Hoop about any diameter: $I = \dfrac{1}{2}MR^2$; same as solid cylinder/disk about C.A.
  - Annular cylinder/ring about C.A: $I = \dfrac{1}{2}M(R_1^2 + R_2^2)$
  - Solid cylinder/disk about central diameter: $I = \dfrac{1}{4}MR^2 + \dfrac{1}{12}M\ell^2$
  - Thin rod about central diameter ($\perp$): $I = \dfrac{1}{12}M\ell^2$
  - Thin rod about end ($\perp$): $I = \dfrac{1}{3}M\ell^2$
  - Solid sphere about any diameter: $I = \dfrac{2}{5}MR^2$
  - Thin spherical shell about any diameter: $I = \dfrac{2}{3}MR^2$
  - Slab about $\perp$ axis through center: $I = \dfrac{1}{12}M(a^2 + b^2)$
- Parallel-Axis Theorem: $I_{\parallel-\mathrm{axis}} = I_\mathrm{CM} + md^2$
- Work:[†] $W_\mathrm{AB} = \displaystyle\int_{\substack{\mathrm{path}\\ \mathrm{A}\to\mathrm{B}}} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}} = K_\mathrm{B} - K_\mathrm{A}$
- Power: $P = \dfrac{dW}{dt}$

[†]Also applicable to fixed-axis rotating systems with appropriate substituions of $\vec{\boldsymbol{\theta}}$, $\vec{\boldsymbol{\omega}}$, $\vec{\boldsymbol{\alpha}}$, I, $\vec{\boldsymbol{\tau}}$, and/or $\vec{\mathbf{L}}$.

- Kinetic energy of particle:† $K = \frac{1}{2}mv^2$
- Average kinetic energy of hot particles: $\bar{K} \approx k_\mathrm{B}\,T$
- Gravitational potential energy: $U(r) = -\frac{Gm_1m_2}{r}$
- Potential energy of particle near Earth's surface: $U(y) = mgy + \text{constant}$
- Mass-energy equivalence: $E = mc^2$
- Period (T)-frequency (f) relation: $T = \frac{1}{f}$
- Wavelength (λ)-frequency relation: $v = \lambda f$
- General 1D wave equation: $x(t) = A\cos(\omega t + \phi)$
- Doppler shift: $\frac{v}{c} = \frac{\lambda_\mathrm{obs} - \lambda_\mathrm{em}}{\lambda_\mathrm{em}}$
- Photon energy: $E = hf$
- Newton's generalization of Kepler's 3rd Law: $T^2 = \frac{4\pi^2}{G(M+m)}\,a^3$, where a is total distance between centers of M and m.

Vectors

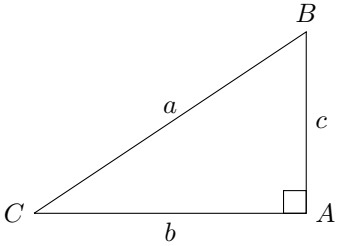
- Direction convention: $\vec{\mathbf{r}}_{12}$ points from 1 to 2
- Unit vector $\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}$
- Scalar/dot product: $\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = AB\cos\theta$, where θ is angle between, or: $\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = \sum_{u\ \text{comp.}} A_u B_u$
- Vector/cross product: $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = AB\sin\theta$, in direction $\perp$ to $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, or: $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \sum_{u\ \text{comp.}} \sum_{w\ \text{comp.}} A_u B_w (\hat{\mathbf{u}} \times \hat{\mathbf{w}})$

Geometry

- Radian: $2\pi\,\text{rad} = 360°$
- Circumference of a circle: $s = 2\pi r$
- Area of a circle: $A = \pi r^2$
- Area of a triangle: $A = \frac{1}{2}bh$
- Area of a trapezoid: $A = \frac{1}{2}(a+b)h$
- Surface area of a sphere: $A = 4\pi r^2$
- Volume of a sphere: $V = \frac{4}{3}\pi r^3$
- Surface area of a cylinder: $A = 2\pi rh + 2\pi r^2$
- Volume of a cylinder: $V = \pi r^2 h$
- Surface area of a cone: $A = \pi r\left(r + \sqrt{h^2 + r^2}\right)$
- Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Trigonometry

- $a^2 = b^2 + c^2$
- $\angle A + \angle B + \angle C = 180° = \pi\,\text{rad}$
- $\sin(\angle B) = \frac{1}{\csc(\angle B)} = \cos(\angle C) = \frac{b}{a}$
- $\cos(\angle B) = \frac{1}{\sec(\angle B)} = \sin(\angle C) = \frac{c}{a}$
- $\tan(\angle B) = \frac{1}{\cot(\angle B)} = \frac{b}{c} = \cot(\angle C)$ (discontinuous function)
- $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$
- $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$



Calculus

- $\frac{d}{dx}f(x) \equiv \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$
- Chain rule: $\frac{d}{dx}f(y) = \frac{df(y)}{dy}\frac{dy}{dx}$
- Product rule: $\frac{d}{dx}\big(f(x)g(x)\big) = f(x)\frac{dg(x)}{dx} + g(x)\frac{df(x)}{dx}$
- $\frac{d}{dx}\left(x^m\right) = mx^{m-1}$
- $\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$
- $\frac{d}{dx}\left(\sin x\right) = \cos x$
- $\frac{d}{dx}\left(\cos x\right) = -\sin x$
- $\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$
- $\int_{x_0}^{x+\mathcal{N}\Delta x} f(x)dx \equiv \lim_{\Delta x \to 0}\left(\sum_{j=1}^{\mathcal{N}} f(x_0 + j\Delta x)\Delta x\right)$
- $\int_{x_\mathrm{A}}^{x_\mathrm{B}} \frac{d}{dx}f(x) = f(x_\mathrm{B}) - f(x_\mathrm{A})$

Common prefixes

- Giga = 10^9 or billion; denoted as G.
- Mega = 10^6 or million; denoted as M.
- Kilo = 10^3 or thousand; denoted as k.
- Centi = 10^{-2} or one-hundredth; denoted as c.
- Milli = 10^{-3} or one-thousandth; denoted as m.
- Micro = 10^{-6} or one-millionth; denoted as μ and sometimes called "micron" when applied to meters.
- Nano = 10^{-9} or one-billionth; denoted as n.
- Femto = 10^{-15} one-billiardth; denoted as f.

Greek letters (lower,upper case):

α, A	alpha	ν	nu
β, B	beta	ξ, Ξ	xi
γ, Γ	gamma	o, O	omicron
δ, Δ	delta	ϕ, Π	pi
$(\epsilon, \varepsilon), E$	epsilon	$(\rho, \varrho), P$	rho
$z\eta, Z$	zeta	σ, Σ	sigma
η, H	eta	τ, T	tau
$(\theta, \vartheta), \Theta$	theta	υ, Υ	upsilon
ι, I	iota	$(\phi, \varphi), \Phi$	phi
κ, K	kappa	χ, X	chi
λ, Λ	lambda	ψ, Ψ	psi
μ, M	mu	ω, Ω	omega

Non-SI units

- Inch: $1\,\text{in} = 2.540\,\text{cm}$
- Foot: $1\,\text{ft} = 12\,\text{in} = 0.305\,\text{m}$
- Mile: $1\,\text{mi} = 5280\,\text{ft} = 1.609\,\text{km}$
- Pound: $1\,\text{lb} = 16\,\text{oz} = 0.454\,\text{kg}$
- Gallon: $1\,\text{gal} = 128\,\text{fl oz} = 3.785\,\text{L}$