

# TRIGONOMETRY



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2018

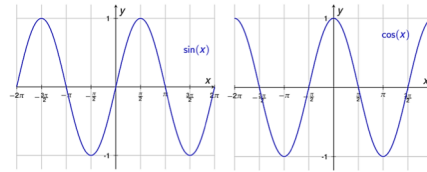
## Degrees to Radians Formula

If  $x$  is an angle in degrees and  $t$  is an angle in radians, then

$$\frac{\pi}{180} = \frac{t}{x}$$

$$t = \frac{\pi x}{180} \quad x = \frac{180t}{\pi}$$

**NOTE:** Trigonometric functions are **periodic**, in that they repeat exactly in regular cycles. The length of the cycle is called a **period**

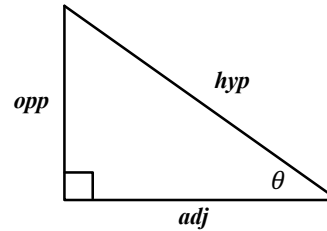


## Periodic Formulas

$$\sin(\theta + 2\pi n) = \sin \theta$$

$$\cos(\theta + 2\pi n) = \cos(\theta)$$

$$\tan(\theta + \pi n) = \tan \theta$$

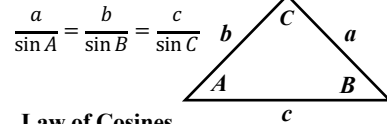


$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \theta = \sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right)$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \theta = \cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right)$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} \quad \theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)$$

## Law of Sines



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

## Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

## Half Angle Identities

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

## Double Angle Identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

## Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

## Even and Odd Identities

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

## Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

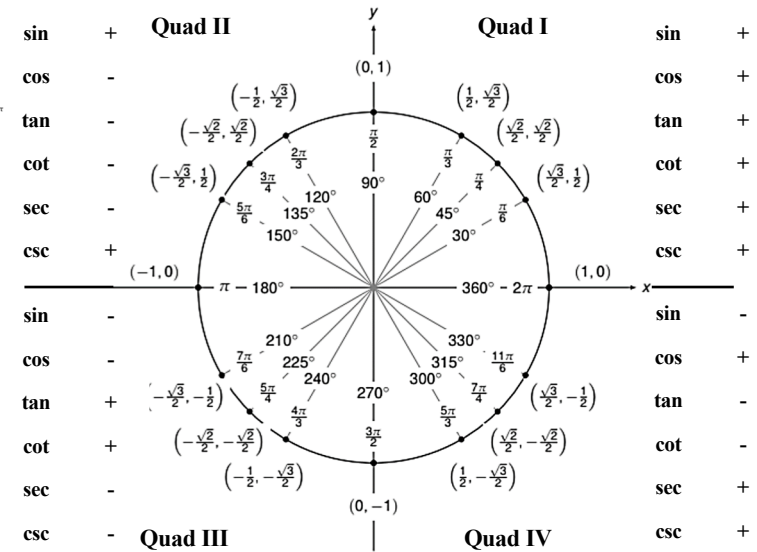
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

## Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$



Degrees	Radians	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\cot \theta$	$\sec \theta$	$\csc \theta$
0	$0\pi$ OR $2\pi$	0	1	0	--	1	--
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90	$\frac{\pi}{2}$	1	0	--	0	--	1
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
135	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
180	$\pi$	0	-1	0	--	-1	--
210	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2
225	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$
270	$\frac{3\pi}{2}$	-1	0	--	0	--	-1
300	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$
315	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
330	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2

## Difference Formulas

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \cos \theta = \frac{1}{\sec \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

## Product to Sum Identities

$$\cos(a) \cos(b) = \frac{1}{2} (\cos(a + b) + \cos(a - b))$$

$$\sin(a) \sin(b) = \frac{1}{2} (\cos(a - b) - \cos(a + b))$$

$$\sin(a) \cos(b) = \frac{1}{2} (\sin(a + b) + \sin(a - b))$$