CALCULUS I

Definition of a Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If $f(x)$ contains a radical (√), multiply by the conjugate.
Example:

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h}\right)^2 - (\sqrt{x})^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \lim_{h \to 0} \frac{x + h - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+n} + \sqrt{x}\right)} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

If $f(x)$ contains negative powers, find a common denominator and combine.
Example:

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{x+h}{x+h}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} = \lim_{h \to 0} \frac{x - (x+h)}{x(x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} -\frac{h}{x(x+h)} \cdot \frac{1}{h} = -\frac{1}{x(x)} = -\frac{1}{x^2}$$

- **In general, the goal is to get rid of an h in the denominator so that evaluating the limit does not result in a division by zero ($h \to 0$).**
- **A function must be continuous to have a derivative.**

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Brown, M. 2018

General Tips

- Always plug in the value of the limit before doing any algebra
- $f(x)$ will give you the position, $f'(x)$ will give you velocity, and $f''(x)$ will give you acceleration
- If a limit is equal to $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$, use L' Hopital's Rule

Differentiation Formulas

Constant Rule

$$\frac{d}{dx} c = 0$$

Power Rule

$$\frac{d}{dx} x^n = nx^{n-1}$$

Product Rule

$$\frac{d}{dx} f(x) \cdot g(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

Quotient Rule

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$$

Chain Rule

$$\frac{d}{dx} f \circ g(x) = f'\left(g(x)\right) \cdot g'(x)$$

Constant Multiple Rule

$$\frac{d}{dx} c \cdot f(x) = c \cdot f'(x)$$

Sum Rule

$$\frac{d}{dx} f(x) + g(x) = f'(x) + g'(x)$$

Useful derivatives to memorize:

Natural Exponent $\frac{d}{dx} e^x = e^x$

Natural Log $\frac{d}{dx} \ln(x) = \frac{1}{x}$

Square Root of x $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$

Trigonometric

$$\frac{d}{dx} \sin(x) = \cos(x) \qquad \frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x) \qquad \frac{d}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x)\tan(x) \qquad \frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$$