Mathematics 2022-2023 Programmatic Assessment Report

Report prepared by: Grady Weyenberg

Course: MATH 422 Mathematical Statistics
Report Date: June 2023
Mathematics Program Learning Outcome 2: Demonstrate mastery of the core concepts in Ring Theory, Real Analysis, Probability, and Statistics.
Readers: Erica Berstein, Keith Edwards, Aaron Tresham

Rubric used:

Table 4. Scoring Rubric for PLO2.

Score	**Probability computation**	**Statistical Concepts**	**Organization**	**Language & prose**
Beginning 1	Unable to select appropriate computations to carry out.	No familiarity with core concepts.	Ideas and writing are completely unorganized	Prose itself is largely incomprehensible. Lack of attention to basics of English grammar and spelling.
Emerging 2	Selects ineffective and difficult approaches to the problem at hand. Calculations contain significant errors. Incorrect use of notation.	Can partially identify relevant aspects of statistical problems. Has difficulty identifying the correct thing to do in every situation, but is somewhat aware of what they should be looking for.	Some attempt at organizing thoughts in a logical manner.	Some problems with grammar and spelling, but not to such a degree that all meaning is obscured.
Competent 3	Calculations are largely successful. Appropriate notation is used.	Largely familiar with main concepts of statistics and can select appropriate analyses to perform.	Minor organizational problems. Communication is mostly organized in a clear logical progression, though some transitions or details may be awkward or unclear.	Minor problems with grammar and spelling that do not significantly impact readability.
Advanced 4	Identifies and uses clever and effective approaches to problem. Notation and manipulations are clear and correct.	Fluent familiarity with statistical concepts. Can readily apply concepts to novel situations and work through to correct answers.	Arguments/proofs are professionally organized and presented in a way that is especially clear and easy to follow. Effective use of paragraphs, displayed equations, sketches, etc., to enhance audience understanding.	Use of English language and grammar is sophisticated and nearly flawless. Writing is well tailored to the audience and purpose.

This assessment addresses the Mathematics Program Learning Outcome related to mastery of core concepts in major areas of mathematics, in particular the topics of Probability and Statistics.

The artifacts selected for this portion of the assessment are in-class written quizzes/ examinations from the MATH 422: Mathematical Statistics course taught in Spring 2023. This course is the second part of a two-semester sequence in Probability and Statistics and is a regularly offered upper-division course intended for Mathematics majors. This two-course sequence is a specific requirement for the Teaching Track in the program, and although it is not specifically listed as a required course for the General Track, it does fulfill upper-division math elective requirements for the General Track, and is one of the few courses that is regularly offered that does so. As a result most math majors end up taking this course.

The two quizzes selected by the instructor represent the high and low end of general student performance on these types of assessments during the semester. Each Quiz assignment consisted of three questions of increasing difficulty. The first question is intended to be as straightforward as possible, being very similar to examples given in class and in the textbook. The second question will be a bit more technically challenging, but still close to the examples given. The third question on the quiz is the most difficult and is intended to require some extension of the concepts presented in class to a novel situation, or combine several separate ideas into a single question.

Quiz 10 is one of the more abstract and technically challenging sections in the course, covering Maximum Likelihood Estimation, and some theoretical results and calculations related to this class of estimators. The quiz questions require students to correctly write down a moderately complex log-likelihood expression starting from a given mass/density form, and then to compute from that a quantity known as the Fisher Information, a process that requires several calculus techniques, including finding second derivatives, and then determining the expectation of the resulting expression (integration). These results must then be plugged into some further expressions in order to arrive at some theoretical results about the asymptotic behavior of the MLE estimators.

Quiz 12 is on the easier side of the difficulty spectrum for material in this course. Students are presented with a variety of statistical analysis problems, and are asked to identify which technique covered in the unit is the most appropriate analysis for the data and research question. Students are also asked to compute the test statistics for a few simple examples and to describe how to set up a rejection region or obtain a p-value for the result.

Student submissions were scored independently by two faculty readers in the mathematics department. Scores from a third reader were not yet available at the time this report was written. Although the readers were faculty in Math and Computer Science, most do not have special familiarity with Probability and Statistics material. Readers were given a Rubric with three categories: Probability and Computation, Statistical Concepts, Language and Organization. Scores from 1 (Beginning) through 4 (Advanced) were assigned to each submission and

category. The scoring rubric for these assignments is included in Table 4 at the end of this document. In general, a score of 2 (Emerging) indicates partial knowledge of a topic with important gaps remaining, while a score a 3 (Competent) indicates that the attempts at solving the problem are largely appropriate, although there are still some errors or misconceptions that need to be addressed before the student has fully mastered the topic.

The results assigned by the readers were largely in agreement with each other. Average scores are presented in the following table.

Table 2. Summary of student scores for PLO2.

	n	Probability Computations	Statistical Concepts	Language and Organization
Quiz 10	11	1.8	2.14	2.48
Quiz 12	10	3.2	2.83	3.03

As expected based on the materials selected for review, students found the Quiz 10 material quite challenging. The average scores of around 2 points correspond with “emerging” familiarity with the material on the rubric. For Quiz 12, with the more straightforward topics, the students earn scores corresponding to the “competent” descriptors on the rubric.

On Quiz 10, covering the material on Maximum Likelihood Estimators and Fisher Information, many students exhibited difficulties with getting underway on the problems at all. Only a few seemed to have mastered the material sufficiently well to be able to map out the steps required to complete the problems, and to accurately write out the log-likelihood expression from the given density function. Moreover, even the students who did have some idea of what they needed to do generally went on to make computational errors in the computations involving derivatives and expectations, indicating that many students lack fluency with not only the statistics material, but also with calculus and algebraic manipulations when the expressions are not extremely simple. No student was assigned a rubric score of 4 (Advanced) by any reader on this Quiz.

An interesting aspect of the types of questions that appear on Quiz 10, is that the students ought to have been able to accurately predict the questions that would appear with relative ease. There are only about 6 distributions of major importance that had been introduced and used repeatedly in this course and its prerequisite MATH 421. About half of these distributions were used as examples of the relevant techniques in the book and in lectures. Thus, there are only a small number of related problems that remain to be used as assessment material. A thoughtful student, or even one who simply looked through the exercises in the book, would have quickly realized what the remaining open questions were,

and could have easily practiced them all within an hour of study or so. Students are permitted to use their notes on a quiz, so doing this type of preparation would be extremely valuable for them. The fact that not a single student apparently did this indicates a broad lack of application of effective study habits and techniques.

For Quiz 12, student performance was significantly improved. In these problems, the statistical and mathematical tasks required are more straightforward: Selecting the appropriate analytical technique from a menu of things covered in class, and then using numbers given in the problem description to determine the results from the chosen technique. The calculation tasks were much simpler, generally only requiring some arithmetic, instead of the calculus required in Quiz 10. Generally, the students with lower scores received these because they attempted to use a technique inappropriate to the problem description. In contrast to Quiz 10, the scores earned by students were generally higher for the Probability Computation category than for Statistical Concepts, indicating the relative ease of the calculation step on these topics. On this Quiz, unlike Quiz 10, readers assigned numerous students a rubric score of 4 (Advanced), and there were no students assigned scores of 1 (Beginning).

In general, students performed best on questions that required only straightforward application of techniques and mathematical formulae that were presented and used in class. Any problem requiring the students to produce or manipulate a mathematical expression of any complexity presented difficulties for many students. Computation errors sometimes went beyond the usual undergraduate issues, such as sign errors, minor arithmetic mistakes, and the occasional dropped term. In particular, basic struggles with expressions involving logarithms and exponentials were more common than they should be at this level. Overall, the quantity and form of the computational errors indicates that students' ability to engage with the conceptual material is being impeded by a need to exert excessive effort and attention on algebra- and calculus-related manipulations that ought to be mostly automatic by this point in their studies.

Results of the assessment will be presented to the department and discussed at a departmental meeting in Fall 2023.